MT 007.02 / SL 266.01
Ideas in Mathematics: The Grammar of Numbers

More sums

1. In class, we showed that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

Show that this formula can also be written as

$$\frac{n\left(n+\frac{1}{2}\right)(n+1)}{3}.$$

2. We can now use this formula along with the formula

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

and basic properties of sigma notation to add more complicated formulas. For example, do each of these sums using the formulas, and then check by writing out the terms and adding them up.

$$\sum_{t=1}^{5}(t^2+t) \qquad \sum_{t=1}^{4}(3t^2+2t+1)$$

3. Compute the following sums (you should use the formula for the left-hand column):

$$\sum_{q=1}^{2} q \qquad \sum_{q=1}^{2} q^3$$

$$\sum_{q=1}^{3} q \qquad \sum_{q=1}^{3} q^3$$

$$\sum_{q=1}^{4} q \qquad \sum_{q=1}^{4} q^3$$

$$\sum_{q=1}^{5} q \qquad \sum_{q=1}^{5} q^3$$

Compute

$$\sum_{j=1}^{100} j$$

and then make a guess about the value of

$$\sum_{j=1}^{100} j^3.$$
