MT00702/SL26601
Ideas in Mathematics: The Grammar of Numbers
Final Examination
May 4, 1998

You must be explicit in discussing how you arrived at your solutions. Little or no credit will be given for solutions without explanation.

Do all of your work in the blue booklets, copying any tables into the books. Please label all of your answers clearly.

Cheating will be severely punished.

1. *(5 points)* Using the Indian *nikhilam* method to multiply two numbers, compute the following product:

	( )	
	97	—
$\times$	81	—
		|

2. *(15 points)*
(2a) Explain how to multiply 27×6 on a Japanese *soroban*.
(2b) How does one add $6 + 8$ on a Chinese *suàn pán*?
(2c) Demonstrate that this particular addition is easier to perform on a *suàn pán* than on a *soroban*.

3. *(5 points)* Remember that the symbol $\overline{X}$ means the complement of X. The complement in this problem means that we write X as a 4-bit numeral, and then take the complement of each bit. In other words, the complement of 0100_2 is 1011_2, *i.e.*, we "flip" the bits.

Suppose that you were asked to construct tables giving values of $X \vee \overline{X}$ and for $X \wedge \overline{X}$, for X running from 0000_2 to 1111_2. Why would you be able to complete these tables almost instantly?

4. *(5 points)* Write $\frac{2}{5}$ as a decimal in base 2.

5. *(15 points)*
(5a) Explain how you would multiply 6×8 using the finger multiplication methods in Menninger.
(5b) Give an algebraic proof that this method always works.

6. *(5 points)* In Japan (and many other East Asian countries), music employs a *pentatonic* scale, containing the notes C-D-F-G-A-c. We will assume that this scale is in natural tuning.
(6a) Why is this scale called pentatonic?
(6b) Using the ratios for an octave and a fifth, both of which you already know, calculate the ratios for the following intervals, showing how you derived them: C-c, C-G, C-F, and C-D.

7. *(10 points)* "Upstairs, downstairs" [based on a section in Adler, *Thinking Machines*]:

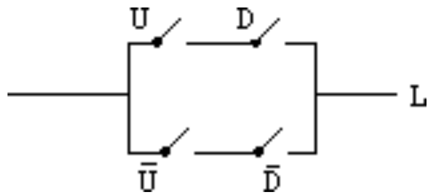
In 1938, Claude Shannon demonstrated that switching circuits can be analyzed by binary (Boolean) algebra. Here is a common switching problem where this applies:

We would like to turn a light L on ($L = 1$) and off ($L = 0$) by using *either* an upstairs switch U or a downstairs switch D such that flipping either switch should change the state of L from on to off, or off to on. We assume that when **both** switches are off (meaning that $D = 0$ and $U = 0$), then the light is off (so that $L = 0$).

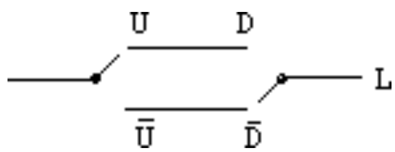
(7a) Why would a parallel circuit ($U \vee D$) not work?

(7b) Why would a series circuit ($U \wedge D$) not work?

The proper solution looks like this:



simplifiable to



and the resulting formula is $L = (U \wedge D) \vee (\overline{U} \wedge \overline{D})$.

(7c) What is the truth table for this circuit?

L		$D=0$	$D=1$
$U=$	0		
	1		

Hint: Rather than working from the logical formula, just think about when the light is on and when it is off.

(7d) What other logical operator(s) does this diagram remind you of?

8. *(5 points)* Compute

$$\sum_{t=1}^{10} 7t^2 + 5t$$

using the formulas discussed in class.

9. *(5 points)* Use the Indian *yāvadūnam* method to compute the square of 999.

10. *(10 points)* Consider the following selected list of Tibetan numerals:

ǰig	1
ǰu	10
ǰugǰig	11
shi	4
ǰubshi	14
shibǰu	40
ña	5
ǰuña	15
ñabǰu	50
gu	9
ǰurgu	19
gubǰu	90

Here are two possible ways to explain the numerals in the Tibetan corpus above.

Solution A

1 = gǰig
4 = bshib
5 = ñab
9 = rgub
10 = ǰu

Rule **A1**: A word may only begin with a single consonant (not counting *h*), so delete the first of two consonants in word-initial position.

Rule **A2**: A word may not end with *b*, so delete word-final *b*.

Solution B

1 = gǰig
4 = bshi
5 = ña
9 = rgu
10 = bǰu

Rule **B1**: A word may only begin with a single consonant (not counting *h*), so delete the first of two consonants in word-initial position.

Which solution is better and why? You will probably have to apply the rules to the combinations of forms in order to evaluate the two solutions. At any rate you must reason, with examples, to justify your decision.

11. *(5 points)* How many perfect order 3 magic squares are there? List all of them, and explain how you know that you have a complete list.

12. *(5 points)* Construct a perfect order 4 magic square.

13. *(5 points)* As we did in class, call $J(n)$ the solution of the Josephus problem, in which alternating people in a circle of n people kill themselves starting with person number 2. What is $J(1025)$? Explain your answer carefully. *Hint:* $1024 = 2^{10}$.

14. *(10 points)* The Indo-European word for 100 is **ḱmtóm.*
(14a) What process has been applied to give the initial *h* of the English word *hundred*?
(14b) What process has been applied to give the initial consonant sound of the Italian word *cento* (where ⟨c⟩ is [ch]).
(14c) Why does the Sanskrit word *śatám* '100' begin with a voiceless palatal fricative [ç]?
(14d) What process explains the final *a* in *śatám*?

15. *(5 points)* Convert 10101_2 to hexadecimal. Convert $\mathrm{E2}_{16}$ to binary.

16. *(5 points)* Prove that $\sqrt{6}$ is irrational. *Hint:* Use the same even-odd argument that showed that $\sqrt{2}$ is irrational.