MT00702/SL26601
Ideas in Mathematics: The Grammar of Numbers
Quiz: April 1, 1998

Name_____

You have 20 minutes to do this quiz. Each question is worth 25 points.

You must be explicit in discussing how you arrived at your solutions. Little or no credit will be given for solutions without explanation.

Cheating will be severely punished.

1. While studying the number 5, we came across an Indian computational technique called *ekādhikena pūrveṇa*, 'by one more than the previous,' which enabled us to calculate rapidly the square of numbers ending in 5. That sûtra also has a sub-corollary: *antyayor daśaka*, 'of two extremes in ten,' which applies for multiplying two numbers whose last digits sum to 10 and whose first digits are equal.

Choose one (and only one) of the following two calculations and use the appropriate sûtra to solve it. Make certain that your work is explicit and demonstrates clearly that you understand how the sûtra works:

1a) 72×78.

1b) 105^2.

2. Compute the sum

$$\sum_{r=1}^{9} r^2 + 2r$$

by using the formulæ that we discussed in class.

3. In class, we identified a number of linguistic processes which apply when languages change over time (historical linguistics). Select **4** (and only 4) of the following examples and identify by name/label the relevant process(es). If you are able to add comment or detail to any example, you should do so:

3a) The consonant change in IE **penk^{w}e* > Sanskrit *pañca* 'five'

3b) The vowels in IE **penk^{w}e* > Sanskrit *pañca* 'five'

3c) The initial consonant in IE **tria* > Gothic *θrija* 'three'

3d) The final consonant in IE **penk^{w}e* > Gothic *fimf* 'five' [two processes!]

3e) The consonant change in Latin *octo* > Italian *otto* 'eight'

4. Answer **both** of the following questions. Provide explicit calculations.

4a) You have landed a cushy job as business manager for the Bergen County Symphony Orchestra. In one wacky modern piece, the composer has specified that the B keys on a harpsichord must be tuned naturally and not in equal temperament like the rest of the instrument.

The professional keyboard tuner for the orchestra doesn't know anything about natural tuning, so you step in and put your credibility on the line by offering to help out with what you remember from some stupid course taught by two lunatics back at Boston College.

You remember (who could ever forget?) that the ratio for a major second in natural tuning is $9 : 8$ and that the ratio for a minor second is $10 : 9$. You also remember how to add intervals.

If **a'** (that is, a above middle c) is tuned at 444Hz, what frequency will you tell the tuner to use on his meter in order to get a naturally tuned **b'**? *Note:* After the tuner gets that one **b'** tuned, he can tune all the other bs on the keyboard by adding and subtracting octaves. You need not do this part of the computation.

4b) If, in Pythagorean tuning, I subtract an interval **c-e** from an octave **c-c'**, what is the ratio of the remaining interval? How does this remaining interval differ from its counterpart in natural (pure) tuning?