## Mathematics 102

Examination 1
September 28, 2004
Name
Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. No credit will be given for answers without explanations. When you are finished with the examination, please fold this piece of paper in half and put it in the blue booklet.

Cheating will result in a failing grade.
The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

Calculators are not permitted on this examination.

1. (16 points) (a) State the Intermediate Value Theorem.
(b) Use the Intermediate Value Theorem to explain why the equation $\sin x=x^{2}-1$ has a solution with $x$ between 0 and $\pi$.
2. (10 points) Find the equation of the tangent line to the graph of $y=3 x^{2}+4$ at the point $(2,16)$.
3. (54 points) Compute the following limits. Be sure to justify your answers. If a limit does not exist, but equals $\infty$ or $-\infty$, you must say so in order to get full credit. As usual, $[x]$ refers to the greatest-integer function.
(a) $\lim _{x \rightarrow 0} \frac{(3+x)^{2}-9}{x}$
(b) $\lim _{x \rightarrow 3^{+}} \frac{x^{2}+2}{x-5}$
(c) $\lim _{x \rightarrow 3^{-}} \frac{x}{[x]}$
(d) $\lim _{x \rightarrow 6^{+}}[2 x+0.99]$
(e) $\lim _{x \rightarrow 4^{-}} \frac{x}{x-4}$
(f) $\lim _{x \rightarrow 5} \frac{x^{2}+9}{x-5}$
(g) $\lim _{x \rightarrow 0} \frac{\sqrt{x^{2}+4}-2}{x^{2}}$
(h) $\lim _{x \rightarrow \frac{\pi}{2}-} \tan x$
(i) $\lim _{x \rightarrow 2} \frac{x^{3}-4 x^{2}+7 x-6}{x-2}$
4. (20 points) The first graph below is the graph of $y=f(x)$, and the second is the graph of $y=g(x):$



Find numbers $A, B, C$, and $D$ so that $g(x)=A f(B(x+C))+D$.

