

Mathematics 102
Examination 2
October 22, 2004

Name _____

Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. No credit will be given for answers without explanations.

Cheating will result in a failing grade.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

Calculators are not permitted on this examination.

1. (10 points) (a) State the definition of $f'(x)$, the derivative of $f(x)$.
(b) Let $f(x) = (x+1)^3 + 4$. Using only the definition of the derivative and basic theorems about limits, compute $f'(4)$.

2. (30 points) Compute $\frac{dy}{dx}$ for each of the following functions.

(a) $\sqrt{x^2 + 1}$ (b) $\tan(2x)$ (c) $(x^3 + 2)^4$
(d) $x \sin 3$ (e) $\sin(3x)$ (f) $3 \sin x$

3. (5 points) State the Intermediate Value Theorem.

4. (10 points) Let $f(x) = |x - 2| + 3$. For which values of x is $f'(x)$ undefined? At which values of x is the function $f(x)$ differentiable? At those values for which the derivative exists, compute $f'(x)$ and simplify your answer as much as possible.

5. (10 points) Consider the graph defined by the equation $x^4 + xy + y^5 = 19$. Find the equation of the tangent line to this graph at the point $(2, 1)$.

6. (20 points) Compute the following limits.

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2} \quad \lim_{h \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} - h\right)}{2h} \quad \lim_{y \rightarrow 0} \frac{\sin y}{\sin 2y} \quad \lim_{x \rightarrow 2^-} [94x - 1]$$

Remember that as usual, $[x]$ refers to the greatest integer function.

7. (15 points) Let n be a positive integer. In class, we derived the formula

$$\frac{d}{dx} (\sin^n x \cos nx) = n \sin^{n-1} x \cos(n+1)x.$$

Find a similar formula for

$$\frac{d}{dx} (\sin^n x \sin nx).$$

Be sure to simplify your answer as much as possible.