Mathematics 102
Examination 3
November 17, 2004

Name______________________________

Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. No credit will be given for answers without explanations.

Cheating will result in a failing grade.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

Calculators are not permitted on this examination.

1. *(5 points)* State the Intermediate Value Theorem.

2. *(10 points)* Find the maximum and minimum values of the function $f(x) = 3x^2+24x+11$ if $0 \leq x \leq 1$.

3. *(15 points)* (a) State the Mean Value Theorem.

(b) If a and b are numbers so that $-\frac{\pi}{2} < a < b < \frac{\pi}{2}$, show that

$$\tan b - \tan a \geq b - a.$$

Hint: Apply the Mean Value Theorem to the function $\tan x$.

4. *(10 points)* Use a linear approximation to give a fractional approximation of $\sqrt{82}$.

5. *(10 points)* Show that the equation $x^{49} + x^{25} + x + 1 = 0$ has exactly one real solution.

6. *(15 points)* Compute the following limits. Be sure to justify your answers. If a limit does not exist, but equals ∞ or $-\infty$, you must say so in order to get full credit. As usual, $[x]$ refers to the greatest-integer function.

$$\lim_{y \to 0} \frac{\tan y}{y} \qquad \lim_{x \to -\infty} \frac{2x^2+3}{5x^3} \qquad \lim_{x \to 3^+} [x] - [-x]$$

7. *(10 points)* Suppose that a rectangle has area A. In terms of the number A, what is the minimum length of the diagonal of the rectangle? *Note:* You do not need to use the second derivative test to show that you have found a minimum and not a maximum.

8. *(25 points)* Let $y = \dfrac{x^2+x-7}{x+8}$. Use the first and second derivatives to figure out the intervals on which y is increasing, decreasing, concave up, and concave down. Find all local extrema and inflection points. Find all vertical and horizontal asymptotes. Show that $y = x - 7$ is a slant asymptote.