

Mathematics 102
Examination 4
December 8, 2004

Name _____

Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. No credit will be given for answers without explanations.

Cheating will result in a failing grade.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

Calculators are not permitted on this examination.

1. (10 points) State the Extreme Value Theorem.
2. (42 points) Compute $\frac{dy}{dx}$ for each of the following functions. You do not need to simplify your answers.

$$\begin{array}{lll} (a) \ y = 2^x & (b) \ y = \log_3 x & (c) \ y = \frac{(x-1)^2(x-2)^3}{\sqrt{x^2+5}} \\ (d) \ y = (\sec^2 x)^x & (e) \ y = \arcsin(x^2+1) & (f) \ y = \sinh(e^x) \end{array}$$

3. (18 points) Compute the following limits. If a limit does not exist, but is equal to $+\infty$ or $-\infty$, you must state that in order to receive full credit.

$$\lim_{x \rightarrow 0^+} \ln(\tan x) \quad \lim_{x \rightarrow 0^+} \ln(\cos x) \quad \lim_{x \rightarrow \infty} \arctan(2x+1)$$

4. (10 points) Let $f(x) = x^2 - 4$. Starting with the value $x_1 = 1$, perform two iterations (steps) of Newton's method to find a solution to the equation $f(x) = 0$. Leave your answer in fractional form.
5. (10 points) Suppose that $g(x) = f^{-1}(x)$, and further that $f(2) = 3$, $f(3) = 4$, $f'(2) = 11$, $f'(3) = 5$, and $f'(4) = 6$. Do you have enough information to compute $g'(3)$? If so, what is $g'(3)$? If not, what additional information do you need to compute $g'(3)$?
6. (10 points) Prove the identity

$$2 \arcsin x = \arccos(1 - 2x^2)$$

if $x \geq 0$. Be sure to point out where in your answer you use the assumption that $x \geq 0$. (The equation is false if $x < 0$, because the left-hand side of the equation will be negative, while the right-hand side is always positive.)