Mathematics 102
Final Examination
December 18, 2004

Name_____

Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. No credit will be given for answers without explanations.

Cheating will result in a failing grade.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

Calculators are not permitted on this examination.

1. *(5 points)* Using only the definition of the derivative and basic facts about limits, compute the derivative of $f(x) = x^{-1/2}$.

2. *(5 points)* State the Intermediate Value Theorem

3. *(12 points)* Compute $\frac{dy}{dx}$ for each of the following functions. You do not need to simplify your answers.

(a) $y = \tan(e^x)$ (b) $y = \log_x 2$ (c) $y = \sqrt{\dfrac{x^2+1}{(x-1)^2}}$

(d) $y = x^x$ (e) $y = \arctan(x^2+1)$ (f) $y = e^{\cosh x}$

4. *(10 points)* Find the equation of the tangent line to the graph defined by the equation $x^2(x^2+y^2) = y^2$ at the point $(\sqrt{2}/2, \sqrt{2}/2)$. Simplify your answer as much as possible.

5. *(10 points)* (a) State the Mean Value Theorem.
(b) Suppose that $-\frac{\pi}{4} < a < b < \frac{\pi}{4}$. Show that

$$\tan b - \tan a < 2(b-a).$$

6. *(5 points)* Find the maximum and minimum values of the function $f(x) = x^3 + 3x + 4$ if $-1 \le x \le 1$.

7. *(8 points)* Compute the following limits. If a limit does not exist, but is ∞ or $-\infty$, you must say to in order to receive full credit.

$$\lim_{x\to 0} \cos\left(\frac{1}{x}\right) \qquad \lim_{x\to\infty} \cos\left(\frac{1}{x}\right) \qquad \lim_{x\to-\infty} \arctan\left(\frac{x}{3}\right) \qquad \lim_{x\to 3} \left[\frac{9x}{5}\right]$$

In the last limit, $\left[\frac{9x}{5}\right]$ refers to the greatest-integer function.

8. *(5 points)* Use a linear approximation to estimate $\sqrt[4]{15}$.

9. *(15 points)* Let $f(x) = 3x + \frac{9}{(x-3)^2}$. Find all vertical, horizontal, and slant asymptotes. Use the first derivative to classify the intervals on which the function is increasing or decreasing, and locate all local extrema. Use the second derivative to classify the intervals on which the graph of the function is concave up and concave down, and locate all inflection points.

10. *(5 points)* The sum of 2 non-negative numbers is 20. Find the numbers if the product of one number and the square root of the other is to be as large as possible.

11. *(10 points)* Suppose that $g(x) = f^{-1}(x)$. Derive formulas for $g'(x)$ and $g''(x)$ in terms of g, f, f', and f''.

12. *(10 points)* Prove the identity

$$\arcsin \frac{x-1}{x+1} = 2 \arctan(\sqrt{x}) - \frac{\pi}{2}.$$