Name__________________________

Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. When you are finished with the exam, please put this examination inside of the blue booklet. No credit will be given for answers without explanations.

Cheating will result in a failing grade.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

Calculators are not permitted on this examination.

1. (10 points) Suppose that

\[
f(x, y) = \int_0^y \cos(x^2 + t^2) \, dt
\]

Compute \( \frac{\partial f}{\partial x}(x, y) \) and \( \frac{\partial f}{\partial y}(x, y) \). Your answers might have definite integrals in them.

2. (30 points) Let \( C \) be a circle of radius 2 with center at the origin, oriented counterclockwise.

(a) Compute

\[
\oint_C x^2 \, dx + x \, dy
\]

directly.

(b) Verify your answer to part (a) by using Green’s Theorem.

3. (30 points) Let \( S \) be the triangle with vertices \((1, 0, 0), (0, 1, 0)\) and \((0, 0, 1)\). The triangle is part of the plane \( x + y + z = 1 \). Verify Stokes’s Theorem for this triangle and the function \( \mathbf{F}(x, y, z) = yi + zk = (y, 0, z) \).

4. (30 points) Let \( D \) be the closed cylinder with boundaries \( z = 1, z = 7, \) and \( x^2 + y^2 = 4 \). Verify Gauss’s Theorem for the solid \( D \) and the function \( \mathbf{F}(x, y, z) = xi + z^2j = (x, z^2, 0) \).