Mathematics 210
Campion 010 — 2 PM
Examination 1
October 1, 2008

Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. Please leave all rational numbers in fractional form. When you are finished with the exam, please put these pages inside of the blue booklet. No credit will be given for answers without explanations.

You may not use calculators.

Cheating will result in a failing grade.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

1. *(15 points)* Finish this definition:

When we say that a set of vectors $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$ is *linearly independent*, we mean that...

2. *(20 points)* (a) Let

$$A = \begin{bmatrix} 2 & 1 & 3 & -2 \\ 2 & 1 & 5 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

What is the reduced row echelon matrix to which A is row equivalent?

(b) Let $T : \mathbf{R}^4 \to \mathbf{R}^3$ be defined by $T(\mathbf{x}) = A\mathbf{x}$. Use your answer to part (a) to determine if T is surjective, injective, both, or neither.

3. *(10 points)* Let

$$A = \begin{bmatrix} 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 0 & 4 \end{bmatrix}.$$

Are the columns of A linearly independent vectors?

4. *(20 points)* Suppose that the set $\{\mathbf{u}, \mathbf{v}\}$ contains 2 linearly independent vectors. Show that the set $\{\mathbf{u}+\mathbf{v}, \mathbf{u}-\mathbf{v}\}$ also contains 2 linearly independent vectors.

5. *(20 points)* Consider this homogeneous system of equations:

$$\begin{aligned} 3x_1 - 2x_2 - x_3 - 4x_4 &= 0 \\ x_1 + x_2 - 2x_3 - 3x_4 &= 0 \end{aligned}$$

Find vectors $\mathbf{u}, \mathbf{v} \in \mathbf{R}^4$ so that the solution can be written in the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = a\mathbf{u} + b\mathbf{v}$ where a and b are arbitrary real numbers.

6. *(15 points)* Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbf{R}^n$. Suppose that $\mathbf{w} = 2\mathbf{u} - 3\mathbf{v}$. Show that the span of the set $\{\mathbf{u}, \mathbf{v}\}$ is the same as the span of the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$.