Mathematics 210
Campion 010 — 2 PM
Examination 2
October 29, 2008

Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. Please leave all rational numbers in fractional form. When you are finished with the exam, please put these pages inside of the blue booklet. No credit will be given for answers without explanations.

You may not use calculators.

Cheating will result in a failing grade.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

1. *(5 points)* Finish this definition:
   When we say that a set of vectors $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$ is *linearly independent*, we mean that...

2. *(15 points)* Compute the matrix product
$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} 9 & 3 & 2 \end{bmatrix}.$$

3. *(10 points)* Let A be an m-by-n matrix. What is the definition of $\operatorname{Nul} A$?

4. *(10 points)* Suppose that A is a square n-by-n matrix, and $\mathbf{x} \in \mathbf{R}^n$ is a non-zero vector so that $A\mathbf{x} = \mathbf{0}$. Find a non-zero square n-by-n matrix B so that $AB = 0$.

5. *(10 points)* Suppose that A is a square $n \times n$ matrix, and $A^2 = 0$. Prove that A is not invertible.

6. *(15 points)* Compute the determinant
$$\begin{vmatrix} 2 & 3 & 4 & 5 & 1 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 3 & 4 & 0 & 0 \\ 1 & 9 & 8 & 0 & 0 \\ 4 & 2 & 3 & 2 & 1 \end{vmatrix}$$

7. *(15 points)* Suppose that $T : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear transformation. Suppose also that $T\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$, and $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$. Find the matrix of T with respect to the standard basis.

8. *(20 points)* Let $H = \operatorname{Span}\left\{\begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}\right\}$. Find a matrix A so that $\operatorname{Nul} A = H$.