Mathematics 210
Campion 010 — 2 PM
Examination 3
November 24, 2008

Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. Please leave all rational numbers in fractional form. When you are finished with the exam, please put these pages inside of the blue booklet. No credit will be given for answers without explanations.

You may not use calculators.

Cheating will result in a failing grade.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

1. *(5 points)* Finish this definition:

   When we say that a set of vectors $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$ is a *basis* of a vector space V, we mean that...

2. *(10 points)* Suppose that $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_5\}$ is a basis for a vector space V. Let $H = \text{Span}\{\mathbf{x}_1 + \mathbf{x}_2, \mathbf{x}_2 + \mathbf{x}_3, \mathbf{x}_1 - \mathbf{x}_3\}$. What is the dimension of H?

3. *(15 points)* Let

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The matrix B is the reduced echelon form of A. (You are not expected to check this statement.)

   (*a*) Find a basis for $\text{Nul}\, A$.
   (*b*) Find a basis for $\text{Col}\, A$.
   (*c*) Find a basis for $\text{Row}\, A$.

4. *(10 points)* Suppose that A is a square matrix, and $A^5 = 0$. Show that the only eigenvalue for A is $\lambda = 0$.

5. *(10 points)* Finish the following definition:

   When we say that a matrix A is *similar* to a matrix B, we mean that...

6. *(10 points)* Show that I, the identity matrix, is similar only to itself.

7. *(20 points)* Let $A = \begin{bmatrix} 7 & -9 \\ 3 & -5 \end{bmatrix}$. Find matrices P and D so that D is diagonal, P is invertible, and $A = PDP^{-1}$.

8. *(20 points)* Remember that $\mathbf{P}_3 = \{a + bt + ct^2 + dt^3\}$, the set of all polynomials of degree at most 3. Define $T : \mathbf{P}_3 \to \mathbf{R}^2$ with the formula $T(p) = \begin{bmatrix} p(2) \\ p(3) \end{bmatrix}$. Find a basis for the kernel of T.