Mathematics 210
Homework 2
Due Friday, September 19, 2 PM

Please note that this homework is due at 2 PM. No late homework can be accepted. You must turn in your answers by the start of class on Friday.

1. Find 4 vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{b} \in \mathbf{R}^3$, all non-zero and all unequal, so that $\mathbf{b}$ is *not* in the span of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

2. Compute

$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ -1 & 11 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

3. Compute

$$\begin{bmatrix} 1 & 2 & 4 \\ 5 & -1 & 11 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}.$$

4. Is $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ in the span of $\begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 11 \\ 9 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 10 \\ 9 \end{bmatrix}$?

5. Can every vector in $\mathbf{R}^2$ be written as a linear combination of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 6 \end{bmatrix}$?

6. Can every vector in $\mathbf{R}^3$ be written as a linear combination of $\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$, $\begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$, and $\begin{bmatrix} 13 \\ 14 \\ 15 \end{bmatrix}$?

7. Let $A = \begin{bmatrix} 2 & 3 & 4 & 7 \\ 12 & 11 & 9 & 6 \end{bmatrix}$. Describe the solution set of the system of equations $A\mathbf{x} = \mathbf{0}$ in parametric form.

The next two problems are designed to prove Theorem 6 in Chapter 1.

8. Suppose that $\mathbf{p}$ solves the equation $A\mathbf{x} = \mathbf{b}$. Suppose that $\mathbf{v}_h$ solves the homogeneous equation $A\mathbf{x} = \mathbf{0}$. Let $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$. Show that $\mathbf{w}$ is a solution of the equation $A\mathbf{x} = \mathbf{b}$.

9. Suppose that $\mathbf{w}$ and $\mathbf{p}$ are two solutions of the equation $A\mathbf{x} = \mathbf{b}$. Let $\mathbf{v}_h = \mathbf{w} - \mathbf{p}$. Show that $\mathbf{v_h}$ solves the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

10. Let

$$A = \begin{bmatrix} 1 & 2 & 11 \\ 3 & 5 & 10 \\ 7 & 6 & 9 \\ 4 & 3 & 2 \end{bmatrix}.$$

Are the columns of A linearly independent vectors?