

Mathematics 210
Homework 3
Due Friday, September 26, 2 PM

Please note that this homework is due at 2 PM. No late homework can be accepted. You must turn in your answers by the start of class on Friday.

1. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 3 \\ 7 & 4 & k \end{bmatrix}$. Find a value of k that makes the columns of A linearly dependent.

2. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 3 \\ 7 & 4 & k \\ 11 & 3 & h \end{bmatrix}$. Find values of h and k that makes the columns of A linearly dependent.

3. (*continued*) Find values of h and k that makes the columns of A linearly independent.

4. Let $A = \begin{bmatrix} 7 & 3 & 2 \\ 2 & 4 & 11 \\ 1 & 1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Is there a vector \mathbf{x} so that $A\mathbf{x} = \mathbf{b}$? If so, is that vector unique?

5. Let $A = \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$, and let $\mathbf{b} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Is there a vector \mathbf{x} so that $A\mathbf{x} = \mathbf{b}$? If so, is that vector unique?

6. Suppose that the set of vectors $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent. Suppose that T is a linear transformation. Show that the set of vectors $\{T(\mathbf{u}), T(\mathbf{v}), T(\mathbf{w})\}$ is linearly dependent.

7. Find a set of linearly independent vectors $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ and a linear transformation T so that the vectors $\{T(\mathbf{u}), T(\mathbf{v}), T(\mathbf{w})\}$ are linearly dependent.

8. Suppose that $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ is a linear transformation defined by the two formulas $T(\mathbf{e}_1) = \mathbf{e}_1$ and $T(\mathbf{e}_2) = 2\mathbf{e}_1 - 2\mathbf{e}_2$. What is the standard matrix of the linear transformation T ?

9. Let $A = \begin{bmatrix} 4 & 6 & 8 \\ 2 & 3 & 4 \end{bmatrix}$. Define a linear transformation $A : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ with the formula $T(\mathbf{x}) = A\mathbf{x}$. Is T onto?

10. (*continued*) Is T one-to-one?