Mathematics 210
Homework 4
Due Friday, October 10, 2 PM

Please note that this homework is due at 2 PM. No late homework can be accepted. You must turn in your answers by the start of class on Friday.

1. Let $A = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$, and $C = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$. Verify that $AB = AC$, even though $B \neq C$.

2. Suppose that A, B, and C are all $n \times n$ matrices. Suppose also that $AB = AC$, but $B \neq C$. Prove that A is not invertible.

3. Find a 2×3 matrix A and a 3×2 matrix B so that $AB = I_2$. Verify that $BA \neq I_3$.

4. Suppose that A is an $n \times m$ matrix and B an $m \times n$ matrix, and that $AB = I$. Let $T_A : \mathbf{R}^m \to \mathbf{R}^n$ be defined by $T_A(\mathbf{x}) = A\mathbf{x}$. Show that T_A is a surjective linear transformation by showing that the equation $A\mathbf{x} = \mathbf{b}$ can be solved for any vector $\mathbf{b}$.

5. (*continued*) Let $T_B : \mathbf{R}^n \to \mathbf{R}^m$ be defined by $T_B(\mathbf{y}) = B\mathbf{y}$. Show that T_B is an injective linear transformation by showing that the equation $B\mathbf{y} = \mathbf{0}$ has only the trivial solution $\mathbf{y} = \mathbf{0}$.

6. Let

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 4 & 3 & -1 \\ 8 & 9 & 2 \end{bmatrix}.$$

Use row reduction to compute A^{-1}.

7. Suppose that A is a 2×2 matrix, that $A \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 17 \\ 20 \end{bmatrix}$, and that $A \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 22 \\ 22 \end{bmatrix}$. Find the matrix A.

8. Suppose that A and B are 2×2 matrices, that $AB = \begin{bmatrix} 2 & 11 \\ 4 & 5 \end{bmatrix}$, and that $B = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$. Find A.

9. Let $A = \begin{bmatrix} 3 & 4 & 5 & 2 \\ 1 & 3 & 2 & 3 \\ 3 & 2 & 1 & 9 \end{bmatrix}$ and let $B = \begin{bmatrix} 6 & 3 \\ 2 & 4 \\ 8 & 3 \\ 1 & 7 \end{bmatrix}$. What is AB?

10. Find a 2×2 matrix A, which does not contain the number 0 in any of its 4 positions, so that A^2 is a matrix containing 4 zeroes.