Mathematics 210
Homework 5
Due Friday, October 17, 2 PM

Please note that this homework is due at 2 PM. No late homework can be accepted. You must turn in your answers by the start of class on Friday.

1. Let $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 8 \\ 6 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$. Let $\mathbf{p} = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$. Determine whether or not $\mathbf{p}$ is in the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

2. Let $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 0 \\ 6 \\ 9 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 8 \\ 6 \\ 4 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 4 \\ 0 \\ 3 \\ 6 \end{bmatrix}$. Let $\mathbf{p} = \begin{bmatrix} 4 \\ 0 \\ 6 \\ 2 \end{bmatrix}$. Determine whether or not $\mathbf{p}$ is in the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

3. Let $A = \begin{bmatrix} 3 & 4 & 5 & -1 & 2 \\ 2 & 3 & 2 & 3 & 2 \\ 7 & 6 & 6 & 5 & 4 \end{bmatrix}$. Find a basis for $\operatorname{Col} A$.

4. (*Continued*) Find a basis for $\operatorname{Nul} A$.

5. Find a basis for the subspace spanned by $\begin{bmatrix} 1 \\ -1 \\ -2 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ -1 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ -6 \\ 8 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 4 \\ -7 \\ 7 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -8 \\ 9 \\ 5 \end{bmatrix}$.

6. Suppose that $A$ is a $2 \times 3$ and $B$ is a $3 \times 2$ matrix. Prove that $BA$ cannot be invertible.

7. Compute $\begin{vmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & 4 & 1 & 0 \\ 8 & 5 & 6 & 7 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{vmatrix}$.

8. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and let $k$ be any real number. Find a formula relating $\det(kA)$ to $\det A$.

9. Find values of $h$ and $k$ so that $\begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$ is in the span of $\begin{bmatrix} 1 \\ 3 \\ h \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 4 \\ 2 \\ k \end{bmatrix}$.

10. Find a $2 \times 2$ matrix $A$ and a non-zero vector $\mathbf{v} \in \mathbf{R}^2$ so that $\mathbf{v}$ is an element of both $\operatorname{Col} A$ and $\operatorname{Nul} A$.