1. Suppose that $A$ and $B$ are $n$-by-$n$ matrices, and $A$ is singular. Show that both $AB$ and $BA$ must also be singular.

2. Let $P = (a, b)$ and $Q = (c, d)$ be 2 points in the $xy$-plane. Show that the equation

$$\begin{vmatrix} 1 & 1 & 1 \\ x & a & c \\ y & b & d \end{vmatrix} = 0$$

gives an equation for the line passing through $P$ and $Q$.

3. Let

$$T = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}.$$ 

Use row operations to show that $T = (b - a)(c - a)(c - b)$.

4. If $A$ is an invertible matrix, show that

$$\det(A^{-1}) = \frac{1}{\det A}.$$ 

5. Let $V$ be a vector space, and $v \in V$. Show using only the definition of a vector space that $0v = 0$.

6. Let $V$ be a vector space, $0 \in V$, and let $k$ be any real number. Show using only the definition of a vector space that $k0 = 0$.

7. Let $H = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : x + y - z = 0$. Show that $H$ is a subspace of $\mathbb{R}^3$.

8. Let $H = \{p(x) \in \mathbb{P} : p(1) = 0\}$. Decide whether or not $H$ is a subspace of $\mathbb{P}$. Be sure to explain your reasoning fully.

9. Suppose that $H$ and $K$ are subspaces of a vector space $V$. Define

$$H + K = \{w \in V : w = h + k, \ h \in H, \ k \in K\}.$$ 

Show that $H + K$ is a subspace of $V$.

10. Let $H = \begin{bmatrix} c - 4d \\ 2c + d \\ c - d \end{bmatrix} \in \mathbb{R}^3 : c, d \in \mathbb{R}$. Show that $H$ is a subspace of $\mathbb{R}^3$ by finding vectors $u$ and $v$ so that $H$ is the span of $\{u, v\}$. 