Mathematics 210
Homework 6
Due Friday, October 24, 2 PM

Please note that this homework is due at 2 PM. No late homework can be accepted. You must turn in your answers by the start of class on Friday.

1. Suppose that A and B are n-by-n matrices, and A is singular. Show that both AB and BA must also be singular.

2. Let $P = (a, b)$ and $Q = (c, d)$ be 2 points in the xy-plane. Show that the equation

$$\begin{vmatrix} 1 & 1 & 1 \\ x & a & c \\ y & b & d \end{vmatrix} = 0$$

gives an equation for the line passing through P and Q.

3. Let

$$T = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}.$$

Use row operations to show that $T = (b - a)(c - a)(c - b)$.

4. If A is an invertible matrix, show that

$$\det(A^{-1}) = \frac{1}{\det A}.$$

5. Let V be a vector space, and $\mathbf{v} \in V$. Show using only the definition of a vector space that $0\mathbf{v} = \mathbf{0}$.

6. Let V be a vector space, $\mathbf{0} \in V$, and let k be any real number. Show using only the definition of a vector space that $k\mathbf{0} = \mathbf{0}$.

7. Let $H = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbf{R}^3 : x + y - z = 0 \right\}$. Show that H is a subspace of $\mathbf{R}^3$.

8. Let $H = \{p(x) \in \mathbf{P} : p(1) = 0\}$. Decide whether or not H is a subspace of $\mathbf{P}$. Be sure to explain your reasoning fully.

9. Suppose that H and K are subspaces of a vector space V. Define

$$H + K = \{\mathbf{w} \in V : \mathbf{w} = \mathbf{h} + \mathbf{k},\ \mathbf{h} \in H,\ \mathbf{k} \in K\}.$$

Show that $H + K$ is a subspace of V.

10. Let $H = \left\{ \begin{bmatrix} c - 4d \\ 2c + d \\ c - d \end{bmatrix} \in \mathbf{R}^3 : c, d \in \mathbf{R} \right\}$. Show that H is a subspace of $\mathbf{R}^3$ by finding vectors $\mathbf{u}$ and $\mathbf{v}$ so that H is the span of $\{\mathbf{u}, \mathbf{v}\}$.