

Mathematics 210
Homework 7
Due Friday, November 7, 2 PM

Please note that this homework is due at 2 PM. No late homework can be accepted. You must turn in your answers by the start of class on Friday.

1. Suppose that $H = \text{Span} \left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}$. Find a basis for H .
2. (*Continued*) Find a 4-by-4 matrix A so that $\text{Col } A = H$.
3. (*Continued*) Find a 4-by-4 matrix B so that $\text{Nul } B = H$.
4. Let $W_1 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbf{R}^2 : xy = 0 \right\}$. Is W_1 a subspace of \mathbf{R}^2 ? If so, prove your answer; if not, explain why not.
5. Let $W_2 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbf{R}^2 : x + y = 0 \right\}$. Is W_2 a subspace of \mathbf{R}^2 ? If so, prove your answer; if not, explain why not.
6. Recall that $\mathbf{P}_2 = \{at^2 + bt + c\}$, the vector space of all polynomials of degree no more than 2. Define a function $T_1 : \mathbf{P}_2 \rightarrow \mathbf{R}^2$ by $T_1(p) = \begin{bmatrix} p(0) \\ p(1) \end{bmatrix}$. Show that the function T_1 is a linear transformation.
7. (*Continued*) Find a polynomial $f(t) \in \mathbf{P}_2$ so that the kernel of T_1 is $\text{Span}\{f(t)\}$.
8. Now define a function $T_2 : \mathbf{P}_2 \rightarrow \mathbf{R}^2$ with the formula $T_2(p) = \begin{bmatrix} p(0) \\ p'(0) \end{bmatrix}$. (As usual, $p'(0)$ means the derivative of $p(t)$ evaluated at $t = 0$.) Show that T_2 is a linear transformation.
9. (*Continued*) Find a polynomial $g(t) \in \mathbf{P}_2$ so that the kernel of T_2 is $\text{Span}\{g(t)\}$.
10. Now define a function $T_3 : \mathbf{P}_2 \rightarrow \mathbf{R}^2$ by $T_3(p) = \begin{bmatrix} p(0) \\ p(0) \end{bmatrix}$. You may assume that T_3 is a linear transformation; the proof is similar to the proofs for the function T_1 . Find polynomials $h_1(t), h_2(t) \in \mathbf{P}_2$ so that $\{h_1(t), h_2(t)\}$ is a basis for the kernel of T_3 .