Mathematics 210
Homework 8
Due Friday, November 14, 2 PM

Please note that this homework is due at 2 PM. No late homework can be accepted. You must turn in your answers by the start of class on Friday.

1. Let $H_1 = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \in \mathbf{R}^4 : a+b+c=0, a-b+c=0, b+2c=0 \right\}$. Find a basis for H_1 and find the dimension of H_1.

2. Let $H_2 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbf{R}^3 : 3a+2b+c=0 \right\}$. Find a basis for H_2, and find the dimension of H_2.

3. Let $\mathcal{B}_1 = \{1, 2t, 4t^2-2t, 8t^3-12t^2\}$ be a basis for $\mathbf{P}_3$. Find $[5t^2-3t]_{\mathcal{B}_1}$. (This means the coordinate vector of $5t^2-3t$ relative to the basis $\mathcal{B}_1$.)

4. Let $\mathcal{B}_1 = \{1, 2t, 4t^2-2t, 8t^3-12t^2\}$ be a basis for $\mathbf{P}_3$. Let $\mathcal{E} = \{1, t, t^2, t^3\}$. Find the matrices $\underset{\mathcal{B}_1 \leftarrow \mathcal{E}}{P}$ and $\underset{\mathcal{E} \leftarrow \mathcal{B}_1}{P}$.

5. Suppose that $\mathbf{a}_1 = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $\mathbf{b}_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, and $\mathbf{b}_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$. Let $\mathcal{A} = \{\mathbf{a}_1, \mathbf{a}_2\}$ and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$. Find the matrices $\underset{\mathcal{B} \leftarrow \mathcal{A}}{P}$ and $\underset{\mathcal{A} \leftarrow \mathcal{B}}{P}$.

6. The matrix $A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$ has an eigenvalue $\lambda = 3$. Find a basis of the eigenspace for λ.

7. Show that λ is an eigenvalue for A if and only if λ is an eigenvalue for A^T. *Hint:* Consider $A - \lambda I$ and $A^T - \lambda I$.

8. Suppose that A is an n-by-n matrix so that every row adds up to the same number s. Show that s is an eigenvalue for A by finding an eigenvector for A.

9. Suppose that B is an n-by-n matrix so that every column adds up to the same number s. Combine the previous two problems to show that s is an eigenvalue for B.

10. Suppose that A is a matrix, and $A^2 = 0$. Show that the only possible eigenvalue for A is 0.