1. Let \( H_1 = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \in \mathbb{R}^4 : a + b + c = 0, a - b + c = 0, b + 2c = 0 \right\} \). Find a basis for \( H_1 \) and find the dimension of \( H_1 \).

2. Let \( H_2 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^3 : 3a + 2b + c = 0 \right\} \). Find a basis for \( H_2 \), and find the dimension of \( H_2 \).

3. Let \( B_1 = \{1, 2t, 4t^2 - 2t, 8t^3 - 12t^2\} \) be a basis for \( P_3 \). Find \([5t^2 - 3t]_{B_1}\). (This means the coordinate vector of \(5t^2 - 3t\) relative to the basis \(B_1\).)

4. Let \( B_1 = \{1, 2t, 4t^2 - 2t, 8t^3 - 12t^2\} \) be a basis for \( P_3 \). Let \( \mathcal{E} = \{1, t, t^2, t^3\} \). Find the matrices \( P_{B_1 \leftarrow \mathcal{E}} \) and \( P_{\mathcal{E} \leftarrow B_1} \).

5. Suppose that \( a_1 = \begin{bmatrix} 7 \\ -2 \end{bmatrix} \), \( a_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \), \( b_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \), and \( b_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \). Let \( A = \{a_1, a_2\} \) and \( B = \{b_1, b_2\} \). Find the matrices \( P_{B \leftarrow A} \) and \( P_{A \leftarrow B} \).

6. The matrix \( A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix} \) has an eigenvalue \( \lambda = 3 \). Find a basis of the eigenspace for \( \lambda \).

7. Show that \( \lambda \) is an eigenvalue for \( A \) if and only if \( \lambda \) is an eigenvalue for \( A^T \). \textit{Hint:} Consider \( A - \lambda I \) and \( A^T - \lambda I \).

8. Suppose that \( A \) is an \( n \)-by-\( n \) matrix so that every row adds up to the same number \( s \). Show that \( s \) is an eigenvalue for \( A \) by finding an eigenvector for \( A \).

9. Suppose that \( B \) is an \( n \)-by-\( n \) matrix so that every column adds up to the same number \( s \). Combine the previous two problems to show that \( s \) is an eigenvalue for \( B \).

10. Suppose that \( A \) is a matrix, and \( A^2 = 0 \). Show that the only possible eigenvalue for \( A \) is \( 0 \).