Mathematics 210
Homework 9
Due Friday, November 21, 2 PM

Please note that this homework is due at 2 PM. No late homework can be accepted. You must turn in your answers by the start of class on Friday.

1. Let $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Find an invertible matrix P and diagonal matrix D so that $A = PDP^{-1}$. (*Note:* This problem is sufficiently intricate to count as a double problem. Also, all eigenvalues in this problem are whole numbers.)

3. Let $A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{bmatrix}$. Find an invertible matrix P and diagonal matrix D so that $A = PDP^{-1}$. (*Note:* This problem is sufficiently intricate to count as a double problem. Also, all eigenvalues in this problem are whole numbers.)

5. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$. Find an invertible matrix P and diagonal matrix D so that $A = PDP^{-1}$.

6. Suppose that λ is an eigenvalue for an invertible matrix A. Show that λ^{-1} is an eigenvalue for A^{-1}.

7. Recall that a square matrix A is *similar* to a square matrix B if there is an invertible matrix K so that $A = KBK^{-1}$. Suppose that A is similar to B, and B is similar to C. Show that A is similar to C.

8. Show that the 0 matrix (that is, the matrix which contains only the number 0) is similar only to itself.

9. Suppose that A is similar to B. Show that $\det A = \det B$.

10. Suppose that A is an n-by-n matrix with n distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. Prove that $\det A = \lambda_1\lambda_2, \cdots \lambda_n$.