Mathematics 210
Homework 10
Due Friday, December 5, 2 PM

Please note that this homework is due at 2 PM. No late homework can be accepted. You must turn in your answers by the start of class on Friday.

1. Suppose that $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. Show that A is *not* diagonalizable. In other words, show that you cannot write $A = PDP^{-1}$, where D is a diagonal matrix.

2. Suppose that $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & t \\ 0 & 0 \end{bmatrix}$, where t is any non-zero real number. Show that A and B are similar.

3. Suppose that $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Find an invertible matrix P and diagonal matrix D so that $A = PDP^{-1}$.

4. Suppose that $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$. Find an invertible matrix P and diagonal matrix D so that $A = PDP^{-1}$.

5. Suppose that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Suppose that A has 2 unequal eigenvalues λ_1 and λ_2. Show that $a + d = \lambda_1 + \lambda_2$.

6. The *Lucas numbers* are the sequence $L_1 = 1$, $L_2 = 3$, $L_3 = 4$, ..., defined by $L_{n+1} = L_n + L_{n-1}$. Find a formula for the Lucas numbers similar to the formula for the Fibonacci numbers that we found in class.

7. Suppose that a predator-prey problem (akin to the one in Section 5.6 of our text) is modelled using the matrix $A = \begin{bmatrix} 0.4 & 0.3 \\ -p & 1.2 \end{bmatrix}$. Show that if the value of $p = 0.325$, the populations of both the predator and the prey will grow. Find the eventual ratio of predator to prey.

8. (*Continued*) Now suppose that $p = 0.5$ in the matrix in the previous problem. Show that both the predator and the prey will eventually perish.

9. Solve the differential equations

$$\begin{aligned} y_1' &= -2y_1 - 5y_2 \\ y_2' &= \quad y_1 + 4y_2 \end{aligned}$$

with initial conditions $y_1(0) = 3$ and $y_2(0) = 2$.

10. Solve the differential equations

$$\begin{aligned} y_1' &= 3y_1 - 2y_2 \\ y_2' &= 2y_1 + 3y_2 \end{aligned}$$

with initial conditions $y_1(0) = 3$ and $y_2(0) = 2$.