

MT216.03: Introduction to Abstract Mathematics
Examination 1
February 17, 2012

Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. No credit will be given for answers without explanations.

Cheating will result in a failing grade.

Calculators may not be used during this examination.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

1. (20 points) Let d be the greatest common divisor of 24 and 37. Use the Euclidean algorithm to find d , and to find integers a and b so that $24a + 37b = d$.

2. (20 points) Define a sequence of real numbers with the definitions

$$x_1 = 1$$
$$x_n = \sqrt{x_{n-1} + 1}$$

Prove by induction that $x_n < 2$ for all positive integers n .

3. (20 points) Let r and n be non-negative integers. Prove using induction that

$$\sum_{k=0}^n \binom{r+k}{k} = \binom{r+n+1}{n}.$$

4. (20 points) Find four different complex numbers z so that $z^4 = -2$. Express each value of z explicitly in terms of radicals.

5. (20 points) Suppose that k and n are integers, with $n \geq 2$ and $k \geq 0$. Prove that

$$F_n F_{n+k} - F_{n-1} F_{n+k+1} = (-1)^{n+1} F_{k+1}.$$