MT216.03: Introduction to Abstract Mathematics
Examination 2
March 28, 2012

Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. No credit will be given for answers without explanations. Cheating will result in a failing grade.

Calculators may not be used during this examination.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

1. (*15 points*) Let n and k be integers, with $n \geq 0$.
   - (a) Give the complete definition of the binomial coefficient $\binom{n}{k}$.
   - (b) State the binomial theorem.
   - (c) What is the coefficient of x^{18} in $(x+2)^{20}$? State your answer both as a binomial coefficient and as a positive integer.

2. (*20 points*) Let $f : X \to Y$ be a function. Suppose that for all subsets $A, B \subset X$, we know that $f(A \cup B) = f(A) \cup f(B)$. Prove or give a counterexample:
   - (a) f must be a surjection.
   - (b) f must be an injection.

3. (*20 points*) Use the Chinese Remainder Theorem to find the smallest positive integer n so that
   - n has a remainder of 15 when divided by 19.
   - n has a remainder of 12 when divided by 21.
   - n is odd.

4. (*30 points*) Suppose that A and B are sets, and $f : A \to B$ any function. Define a relation on A by saying that if $a_1, a_2 \in A$, then $a_1 \sim a_2$ if $f(a_1) = f(a_2)$.
   - (a) Show that this relation is an equivalence relation.
   - (b) Take the specific example of $A = \mathbf{C}$, $B = \mathbf{C}$, and $f(z) = z^4$. What is the equivalence class of the number $1 + i$ under this equivalence relation?

5. (*5 points*) Suppose that $h : Y \to Z$ is a function, and $B \subseteq Z$. What is the definition of the set $h^{-1}(B)$?

6. (*10 points*) Suppose that ζ is a root of unity.
   - (a) Define what is meant by the *order* of ζ.
   - (b) Suppose that ζ has order 60. What is the order of ζ^{36}?