

MT216.03: Introduction to Abstract Mathematics
Examination 3
April 30, 2012

Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. No credit will be given for answers without explanations. Cheating will result in a failing grade.

Calculators may not be used during this examination.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

1. (20 points) Let $f(x), g(x) \in \mathbf{F}_7[x]$, with $f(x) = 5x^2 + x + 2$ and $g(x) = 2x + 3$. Compute the monic greatest common divisor $d(x)$ of f and g , and find polynomials $a(x), b(x) \in \mathbf{F}_7[x]$ so that $d(x) = a(x)f(x) + b(x)g(x)$.
2. (10 points) What is the remainder when 3^{75} is divided by 36?
3. (10 points) Give an example of a degree 6 polynomial which you can show is irreducible by using Eisenstein's Criterion. Be sure to explain your answer.
4. (20 points) Suppose that $f(x), g(x) \in \mathbf{C}[x]$ and we know that
 - $\deg(f) = \deg(g) = n$, with $n \geq 1$.
 - $f(1) = g(1), f(2) = g(2), \dots, f(n) = g(n)$.
 - $f'(1) = g'(1)$.

Prove or give a counterexample: $f(x) = g(x)$.

5. (20 points) List all eight cubic polynomials in $\mathbf{F}_2[x]$, and indicate which of them are irreducible in $\mathbf{F}_2[x]$. Be sure to explain your answer fully.
6. (20 points) Let $f : \mathbf{Z} \rightarrow \mathbf{Q}$ be defined by the formula $f(n) = \frac{n}{2n^2 - 1}$.
 - (a) Is f a surjective function?
 - (b) Is f an injective function?