Mathematics 216
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Homework 4
Due January 30, 2012

1. Let n be a positive integer. Prove using induction that

$$\lim_{x\to\infty} \frac{x^n}{e^x} = 0.$$

2. The Gamma function is defined by the formula

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,dt$$

for $x \geq 1$. This is an improper integral, and for our purposes, you may assume that the integral converges with $x \geq 1$. Prove that $\Gamma(1) = 1$.

3. Use integration by parts, along with problem 1, to prove that $\Gamma(n+1) = n\Gamma(n)$ if n is a positive integer.

4. Prove using induction that if n is a positive integer, then $\Gamma(n) = (n-1)!$.

5. Let n be a positive integer. Show that

$$\sum_{k=1}^{n} F_k = F_{n+2} - 1.$$