

Mathematics 216  
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Homework 5  
Due February 1, 2012

1. Let  $n$  be a positive integer. Prove using induction that

$$\lim_{x \rightarrow 0^+} x(\log x)^n = 0.$$

The notation  $\lim_{x \rightarrow 0^+}$  means that  $x$  tends to 0 and is positive. The inequality  $x > 0$  is required because  $\log x$  is only defined for positive  $x$ . *Hint:* Apply l'Hôpital's rule, but make sure that you do it correctly.

2. Use an even-odd argument to show that  $\sqrt{13}$  is irrational. *Hint:* This is a bit tricky, and requires a bit more thought than our previous irrationality proofs.

3. Find the smallest positive integer  $N$  so that  $n^3 \leq 2^n$  if  $n \geq N$ , and prove your result using induction.

4. Let  $n > 2$  be an integer. Show that

$$F_n F_{n+1} - F_{n-1} F_{n+2} = (-1)^{n+1}.$$