

Mathematics 216
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Homework 6
Due February 3, 2012

1. Let n be a positive integer. Prove using induction and integration by parts that

$$\int_0^1 (-\log x)^n dx = n!.$$

This is an improper integral, so you will need to explain how you evaluated the lower limit when applying integration by parts.

2. Let m and n be positive integers, with $m \leq n$. Prove that

$$\sum_{k=m}^n \binom{k}{m} = \binom{n+1}{m+1}.$$

3. Let a and b be positive numbers, with $b > 1$. Prove that

$$F_a F_{b-1} + F_{a+1} F_b = F_{a+b}.$$