Mathematics 216
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Homework 7
Due February 6, 2012

1. If $f(x)$ is a function of x, write $f^{(n)}$ to refer to the nth derivative of f with respect to x. If, for example, $f(x) = \sin x$, then $f^{(5)}(x) = \cos x$ and $f^{(6)}(x) = -\sin x$. We define $f^{(0)}(x)$ to be $f(x)$.

Suppose that $u(x)$ and $v(x)$ are functions of x. To save space, we will just write u and v rather than $u(x)$ and $v(x)$. Suppose that n is a positive integer. Prove that

$$(uv)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} u^{(k)} v^{(n-k)}.$$

In other words, you will prove that

$$\frac{d^n}{dx^n}(uv) = \sum_{k=0}^{n} \binom{n}{k} \left(\frac{d^k u}{dx^k}\right) \left(\frac{d^{n-k} v}{dx^{n-k}}\right).$$

Hint: Proceed as in the proof of the binomial theorem.

2. Use the previous formula to compute $\dfrac{d^7}{dx^7}\left(x^2 e^{3x}\right)$.

3. Let n be a positive integer. Show that

$$\sum_{k=1}^{n} F_k^2 = F_n F_{n+1}.$$