

Mathematics 216  
Robert Gross  
Homework 8  
Due February 8, 2012

1. Let  $n$  be an integer which is at least 2. Use integration by parts to derive the formula

$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx.$$

2. Let  $n$  be an integer which is at least 1. Use induction and the previous problem to prove that

$$\int_0^{\frac{\pi}{2}} \cos^{2n+1} x \, dx = \frac{2^{2n}(n!)^2}{(2n+1)!}.$$

3. Let  $n$  be a positive integer. Prove that  $\binom{2n}{n}$  is always even.