

Mathematics 216
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Homework 9
Due February 10, 2012

1. Let n be an integer which is at least 1. Use induction and the previous homework to prove that

$$\int_0^{\frac{\pi}{2}} \cos^{2n} x \, dx = \left(\frac{(2n-1)!}{2^{2n-1} n! (n-1)!} \right) \left(\frac{\pi}{2} \right).$$

2. Find three different complex numbers which solve the equation $z^3 = i$. Express each of the complex numbers in the form $a + bi$, where a and b are real numbers.

3. Let n be a positive integer, and α any non-negative real number. Prove by induction that

$$(1 + \alpha)^n \geq 1 + n\alpha + \frac{n(n-1)}{2} \alpha^2.$$

Be sure in your proof to indicate where you used the fact that $\alpha \geq 0$, because the result is false if α is negative.