

Mathematics 216
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Homework 10
Due February 13, 2012

1. Define the *harmonic numbers* H_n with the formula

$$H_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n} = \sum_{k=1}^n \frac{1}{k}.$$

Prove that

$$\sum_{k=1}^{n-1} H_k = nH_n - n,$$

if $n \geq 2$.

2. Define the Fermat numbers f_n with the formula $f_n = 2^{2^n} + 1$ if $n \geq 0$. The first few Fermat numbers are:

$$\begin{aligned} f_0 &= 3 \\ f_1 &= 5 \\ f_2 &= 17 \\ f_3 &= 257 \\ f_4 &= 65537 \\ f_5 &= 4294967297 \end{aligned}$$

Prove that

$$f_0 f_1 f_2 \cdots f_n + 2 = f_{n+1}.$$

3. Define a sequence of real numbers with the definitions

$$\begin{aligned} x_1 &= 3 \\ x_n &= \sqrt{2x_{n-1} + 1} \end{aligned}$$

If n is a positive integer, prove using induction that $x_n \geq x_{n+1}$.