Mathematics 216
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Homework 12
Due February 22, 2012

1. Suppose that c and d are positive integers. Prove using induction that $F_c | F_{cd}$. The formula $F_a F_{b-1} + F_{a+1} F_b = F_{a+b}$ might be helpful.

2. Suppose that a and b are positive integers, with $d = (a, b)$. Suppose that we have used the Euclidean algorithm to determine integers x and y so that $ax + by = d$. Prove that x and y are relatively prime.

3. Suppose that n and k are positive integers. Prove that

$$\binom{n}{k} = \frac{n}{k} \cdot \binom{n-1}{k-1}.$$

4. Let n be a positive integer. Prove that $\binom{3n}{n}$ is always a multiple of 3.