

Mathematics 216
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Homework 17
Due March 12, 2012

1. Find sets A and B and functions $f : A \rightarrow B$ and $g : B \rightarrow A$ so that f is not a bijection, g is not a bijection, and $f \circ g$ is a bijection.
2. Define $f : \mathbf{R} \rightarrow \mathbf{R} \times \mathbf{R}$ with the formula $f(x) = (x^2, \sin(x))$.
 - (a) Is f an injection?
 - (b) Is f a surjection?
3. Let $\mu_7 = \{\alpha \in \mathbf{C} : \alpha^7 = 1\}$. Define $f : \mu_7 \rightarrow \mu_7$ with the formula $f(\alpha) = \alpha^2$. Is the function f injective, surjective, both, or neither?
4. Now set $\mu_{12} = \{\alpha \in \mathbf{C} : \alpha^{12} = 1\}$. Define $g : \mu_{12} \rightarrow \mu_{12}$ with the formula $g(\alpha) = \alpha^2$. Is the function g injective, surjective, both, or neither?
5. Define a new set operation $A\Delta B$ with the formula

$$A\Delta B = (A \setminus B) \cup (B \setminus A).$$

On the last homework, we saw that $A\Delta B = (A \cup B) \setminus (A \cap B)$. This operation is sometimes called the *symmetric difference* of the sets A and B .

- (a) Show that $A\Delta B = B\Delta A$.
- (b) Show that $(A\Delta B)\Delta C = A\Delta(B\Delta C)$.