Mathematics 216 Robert Gross Homework 18 Due March 14, 2012

1. Find a function $f: A \to B$ and subsets $X, Y \subset A$ so that $f(X \cap Y) \neq f(X) \cap f(Y)$.

2. Suppose that $f: A \to B$ is an injective function, and $X, Y \subset A$. Show that $f(X \cap Y) = f(X) \cap f(Y)$.

3. Let A be the set of non-negative integers. Define a function $f : A \times A \to A$ with the formula $f(a, b) = \begin{pmatrix} a+b\\ a-b \end{pmatrix}$. Is f an injection? Is f a surjection? Be sure to explain your answer fully. Remember that we defined $\begin{pmatrix} r\\ s \end{pmatrix}$ to be 0 if s < 0, so the function definition makes sense.