

Mathematics 216  
Robert Gross  
Homework 20  
Due March 19, 2012

1. Suppose that  $f : X \rightarrow Y$ , and for every set  $A \subseteq X$ ,  $f^{-1}(f(A)) = A$ . Prove that  $f$  is an injection.

2. Let  $\mathbf{Q}^\times$  be the set of all non-zero fractions. Define a relation  $\sim$  on  $\mathbf{Q}^\times$  by saying that  $\frac{a}{b} \sim \frac{c}{d}$  if  $\frac{ad}{bc} = \left(\frac{p}{q}\right)^2$ , where  $\frac{p}{q}$  is a non-zero fraction. For example,  $\frac{3}{4} \sim \frac{16}{3}$ .  
Show that  $\sim$  is an equivalence relation.

3. Let  $n$  be a positive integer. Remember that  $\mu_n$ , the set of  $n$ th roots of unity, is defined by  $\mu_n = \{z \in \mathbf{C} : z^n = 1\}$ . Remember also that if  $z \in \mu_n$ , the *order* of  $z$  is the smallest positive integer  $k$  so that  $z^k = 1$ .

Define a relation  $\sim$  on  $\mu_n$  by saying that  $z \sim w$  if the order of  $z$  and the order of  $w$  are equal.

(a) Show that this is an equivalence relation.

(b) List the equivalence classes in  $\mu_{10}$  under this equivalence relation. How many different equivalence classes are there?