Mathematics 216
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Homework 22
Answers

1. Define a relation on $\mathbf{Z}$ by setting $a \sim b$ if $ab > 0$. Is this an equivalence relation? If so, how many unequal equivalence classes are there?

Answer: This is *not* an equivalence relation. It is not true that $0 \sim 0$, so the relation is not reflexive.

2. Now define a relation on $\mathbf{Z}$ by setting $a \sim b$ if $ab \geq 0$. Is this an equivalence relation? If so, how many unequal equivalence classes are there?

Answer: This is also *not* an equivalence relation. We have $1 \sim 0$, and $0 \sim -1$, but $1 \not\sim -1$, so the relation is not transitive.

3. Use the Chinese Remainder Theorem to find the smallest positive integer n so that

$$n \equiv 23 \pmod{34}$$
$$n \equiv 11 \pmod{23}$$
$$n \equiv 14 \pmod{19}$$

Answer: Start with $n \equiv 23 \pmod{34}$, and rewrite that congruence as the equation $n = 34a + 23$. Substitute into the second congruence, and we have $34a + 23 \equiv 11 \pmod{23}$, or $11a \equiv 11 \pmod{23}$, with the obvious solution $a \equiv 1 \pmod{23}$. That means that $a = 23b + 1$, and so $n = 34a + 23 = 34(23b + 1) + 23 = 782b + 57$. Now, substitute into the third congruence, and we have $782b + 57 \equiv 14 \pmod{19}$, or $3b \equiv -43 \equiv 14 \pmod{19}$. We could use the Euclidean algorithm here, or we could just notice that this is the same as $3b \equiv 33 \pmod{19}$, and so $b \equiv 11 \pmod{19}$. Therefore, $b = 19c + 11$, and $n = 782b + 57 = 782(19c + 11) + 57 = 14858c + 8659$.

Therefore, the smallest positive integer solution to the three congruences is $n = 8659$.