

Mathematics 216
Robert Gross
Homework 23
Due March 26, 2012

1. Decide if $f : \mathbf{Z}/7\mathbf{Z} \rightarrow \mathbf{Z}/14\mathbf{Z}$ given by the formula $f([x]_7) = [x^2]_{14}$ is a well-defined function. Be sure to explain your answer fully.
2. Let n be a positive integer. Show that $g : \mathbf{Z}/2^n\mathbf{Z} \rightarrow \mathbf{Z}/2^{n+1}\mathbf{Z}$ defined by $g([x]_{2^n}) = [x^2]_{2^{n+1}}$ is well-defined.
3. Suppose that A is a finite set, $f : A \rightarrow A$, and $g : A \rightarrow A$. Suppose in addition that $f \circ g : A \rightarrow A$ is a bijection. Prove that f and g are both bijections.
4. Give an explicit example to show that the conclusion to the previous problem is *false* if A is an infinite set. You need to tell me what you are using for the set A , what the functions f and g are, and why neither f nor g are bijections.