1. Define a relation $\sim$ on $\mathbb{R}^2$ by setting $(a, b) \sim (c, d)$ if there is a non-zero real number $\lambda$ so that $(a, b) = (\lambda c, \lambda d)$. Prove that $\sim$ is an equivalence relation. Be sure to explain in your proof where it was important that $\lambda \neq 0$.

2. Suppose that $a$, $b$, and $c$ are positive integers, and $(a, b) = 2$ and $(a, c) = 3$. Say as much as possible about $(a, bc)$.

3. Suppose that $a$, $b$, and $c$ are positive integers, and $(a, b) = 2$ and $(a, c) = 4$. Say as much as possible about $(a, bc)$.

4. Suppose that $a$, $b$, and $c$ are positive integers, and $(a, b) = 2$ and $(a, c) = 4$. Say as much as possible about $(a^2, bc)$. 