1. Let \( f(x) = 2x^2 + 3x + 1 \) and let \( g(x) = 3x^4 + 2x + 1 \). Consider both \( f(x) \) and \( g(x) \) as elements of \( \mathbb{F}_5[x] \), and compute \( q(x) \) and \( r(x) \) so that \( g(x) = q(x)f(x) + r(x) \) with \( \deg(r) < 2 \).

2. Find the greatest common divisor of \( x^5 - 1 \) and \( 2x^2 + 3x + 1 \) as elements of \( \mathbb{F}_{11}[x] \). Then find polynomials \( f, g \in \mathbb{F}_{11}[x] \) so that \( (x^5 - 1)f + (2x^2 + 3x + 1)g = (x^5 - 1, 2x^2 + 3x + 1) \). Remember that the greatest common divisor is defined to be monic.

3. On a previous homework, we defined the concept of similar matrices: \( A, B \in M_2(\mathbb{R}) \) are similar if there is an invertible matrix \( T \) so that \( AT = TB \). Suppose that \( A \) and \( B \) are similar and that \( A \) is invertible. Prove that \( B \) is invertible.

4. Suppose that \( f : \mathbb{Q} \rightarrow \mathbb{Q} \) is defined by \( f(x) = \frac{x}{x^2 - 2} \). Is \( f \) a surjection? Is \( f \) an injection? Be sure to explain your answer.

5. Suppose that \( g : \mathbb{Z} \rightarrow \mathbb{Q} \) is defined by \( g(x) = \frac{x}{x^2 - 2} \). Is \( g \) a surjection? Is \( g \) an injection? Be sure to explain your answer.