Mathematics 216
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Homework 28
Due April 18, 2012

1. Let $f(x) = 2x^2 + 3x + 1$ and let $g(x) = 3x^4 + 2x + 1$. Consider both $f(x)$ and $g(x)$ as elements of $\mathbf{F}_5[x]$, and compute $q(x)$ and $r(x)$ so that $g(x) = q(x)f(x) + r(x)$ with $\deg(r) < 2$.

2. Find the greatest common divisor of $x^5 - 1$ and $2x^2 + 3x + 1$ as elements of $\mathbf{F}_{11}[x]$. Then find polynomials $f, g \in \mathbf{F}_{11}[x]$ so that $(x^5 - 1)f + (2x^2 + 3x + 1)g = (x^5 - 1, 2x^2 + 3x + 1)$. Remember that the greatest common divisor is defined to be monic.

3. On a previous homework, we defined the concept of similar matrices: $A, B \in M_2(\mathbf{R})$ are similar if there is an invertible matrix T so that $AT = TB$. Suppose that A and B are similar and that A is invertible. Prove that B is invertible.

4. Suppose that $f : \mathbf{Q} \to \mathbf{Q}$ is defined by $f(x) = \dfrac{x}{x^2 - 2}$. Is f a surjection? Is f an injection? Be sure to explain your answer.

5. Suppose that $g : \mathbf{Z} \to \mathbf{Q}$ is defined by $g(x) = \dfrac{x}{x^2 - 2}$. Is g a surjection? Is g an injection? Be sure to explain your answer.