

Mathematics 216
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Homework 30
Due April 23, 2012

1. Let p be an odd prime. Show that in $\mathbf{F}_p[x]$, the polynomial $x^{p-1} - 1$ factors as

$$x^{p-1} - 1 \equiv (x - 1)(x - 2)(x - 3) \cdots (x - (p - 2))(x - (p - 1))$$

2. Substitute $x \equiv 0$ into this factorization to derive a congruence involving $(p - 1)!$.
3. Show that in $\mathbf{Z}/12\mathbf{Z}[x]$, there are two different ways to factor the polynomial $x^2 - x$ into linear factors.