Mathematics 216
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Homework 31
Due April 25, 2012

1. Suppose that $f(x), g(x) \in \mathbf{C}[x]$ are two monic polynomials, with $\deg(f) = \deg(g) = n \geq 1$. Suppose also that $f(1) = g(1)$, $f(2) = g(2)$, $\ldots$, $f(n) = g(n)$. Show that $f(x) = g(x)$. *Hint:* Let $h(x) = f(x) - g(x)$. What is the degree of h? What are some of its roots?

2. Suppose that we remove the assumption that $f(x)$ and $g(x)$ are monic in the previous problem. Show by example that we can no longer conclude that $f(x) = g(x)$.

3. Factor $x^4 + x^2 + 1$ into irreducible factors in $\mathbf{Q}[x]$.

4. Factor $x^4 + x^2 + 1$ into irreducible factors in $\mathbf{F}_2[x]$.

5. Factor $x^4 + x^2 + 1$ into irreducible factors in $\mathbf{F}_7[x]$.