Mathematics 235
Examination 3
Answers

1. (*10 points*) Suppose that $f(x,y)$ is a function which has $(2,4)$ and $(3,5)$ as critical points. Suppose that we are also given the information in this table:

x	y	$f_{xx}(x,y)$	$f_{xy}(x,y)$	$f_{yy}(x,y)$
2	4	5	-3	1
3	5	2	-1	2

Use this information and the second derivative test to decide if each of the two critical pairs $(2,4)$ and $(3,5)$ is a local minimum, a local maxima, or a saddle point.

Answer: Remember that $\Delta = f_{xx}f_{yy} - f_{xy}^2$. For the critical pair $(2,4)$, we have $\Delta = -4 < 0$, and therefore $(2,4)$ is a saddle point. For the critical pair $(3,5)$, we have $\Delta = 3$, and $f_{xx} = 2 > 0$, and therefore $(3,5)$ is a local minimum.

2. (*15 points*) Studies have shown that 10% of the American population suffers from a particular form of heart disease, and that 90% do not have this form. One method of diagnosing this type of heart disease is to administer a stress test. A person can have either a positive or a negative stress test. The probability of a positive stress test in a person with this form of disease is 0.96, and the probability of a negative stress test in a person without this form of disease is 0.97.

Suppose that a person is tested at random, and has a positive stress test. What is the probability that the person has this form of heart disease? Do all of your calculations to 4 decimal places.

Answer: Let P stand for a positive stress test, N for a negative stress test, H for heart disease, and H^c for no heart disease. A table of joint probabilities is:

	P	N
H	0.0960	0.0040
H^c	0.0270	0.8730

Therefore, $P(H|P) = 0.0960/(0.0960+0.0270) \approx 0.7805 \approx 78\%$.

3. (*20 points*) **KlipZ Ninety-Nine Cent Store** specializes in inexpensive office supplies. The demand for paper clips is constant throughout the year, and annual sales are 20,000 boxes of paper clips. Each time that KlipZ places an order for paper clips, there is a shipping and handling fee of \$7.50, and the cost of each box of clips is \$0.37. KlipZ is a small store, with limited storage space, and estimates that the annual cost of storing a box of paper clips in inventory is \$0.09.

What order size should KlipZ use to minimize total annual cost? Each order can be no smaller than 1 box, and no larger than 20,000 boxes. The order size does not need to be an integer.

Answer: Let x be the order size. The annual number of orders is $20000/x$. The average number of boxes in inventory is $x/2$. Therefore, the annual inventory cost is $0.09(\frac{x}{2})$. The annual cost for shipping and handling is $7.50(\frac{20000}{x})$. The annual cost of the materials is $20000 \cdot 0.37$.

The total annual cost is

$$C(x) = 20000 \cdot 0.37 + 7.50\left(\frac{20000}{x}\right) + 0.09\left(\frac{x}{2}\right),$$

and so

$$C'(x) = -7.50\left(\frac{20000}{x^2}\right) + \left(\frac{0.09}{2}\right).$$

Solving $C'(x) = 0$ gives $x^2 \approx 3333333.3333$, or $x \approx 1825.7419$.

We can now compute that $C(1825.7419) \approx 7564.3168$, and $C(1) \approx 157400.0450$, and $C(20000) \approx 8307.5000$. Therefore, the order size which minimizes cost is approximately 1825.7419.

4. (*20 points*) Find the maximum and minimum values of the function $f(x,y) = 2x - 3y$ if $x^2 + 2y^2 = 11$. The values of x and y are not restricted, and can be positive and negative.

Answer: We write $g(x,y) = x^2 + 2y^2 - 11$, and we need to simultaneously solve the three Lagrange equations

$$\begin{aligned} \frac{\partial f}{\partial x} &= \lambda \frac{\partial g}{\partial x} \\ \frac{\partial f}{\partial y} &= \lambda \frac{\partial g}{\partial y} \\ g(x,y) &= 0 \end{aligned}$$

We get

$$\begin{aligned} 2 &= \lambda(2x) \\ -3 &= \lambda(4y) \\ x^2 + 2y^2 &= 11. \end{aligned}$$

Divide the first equation by the second, and we have $-\frac{2}{3} = \frac{x}{2y}$, or $3x = -4y$. This is the same as $x = -\frac{4}{3}y$, and substitution into the last equation yields $\frac{16}{9}y^2 + 2y^2 = 11$, or $34y^2 = 99$, and then $y \approx \pm 1.7064$. Because $x = -\frac{4}{3}y$, we know that when $y = -1.7064$, then $x = 2.2752$, and when $y = 1.7064$, then $x = -2.2752$. The maximum value of $2x - 3y$ occurs when x is positive and y is negative, and the minimum value occurs when x is negative and y is positive. The maximum value turns out to be approximately 9.6695, and the minimum value is approximately -9.6695.

5. (*20 points*) **Stanley's Sub Shop** sells food from a truck during the summer. The truck is small, and has a limited amount of shelf space. Stanley knows that the sales of different items depends on the weather, which he classifies as mild (M), hot (H), and rainy (R). Stanley can load his truck each day with one of assortments of food, which he called A and B. The payoff table depends on whether he chooses type A or type B, and also on the weather. His profits are:

	Weather		
Food assortment	*M*	*H*	*R*
A	83	71	52
B	89	46	58

(*a*) If Stanley uses the optimistic approach, which food assortment should he use?
(*b*) If Stanley uses the conservative approach, which food assortment should he use?
(*c*) If Stanley uses the minimax regret approach, which food assortment should he use?
(*d*) Stanley calls the National Oceanographic and Atmospheric Administration, and determines that the probabilities for the 3 types of weather are:

$$P(M) = 0.35 \qquad P(H) = 0.40 \qquad P(R) = 0.25$$

If Stanley uses the Expected Value approach, which food assortment should he stock? What is the Expected Value of Perfect Information?

Answer: (*a*) The optimistic approach calls for food assortment B, because the profit of 89 is the single largest profit in the table.

(*b*) The conservative approach calls for food assortment A, because in the worst circumstances, A has a profit of no worse than 52, whereas B might have a profit of as little as 46.

(*c*) We write out a table of regrets:

	Weather			*Regrets*			
Food assortment	*M*	*H*	*R*	*M*	*H*	*R*	*Maximum*
A	83	71	52	6	0	6	6
B	89	46	58	0	25	0	25

The minimax regrets approach calls for assortment A.

(*d*) The expected value of assortment A is 70.4500. The expected value of assortment B is 64.0500. Therefore, the expected value approach calls for choosing assortment A.

With Perfect Information, we would choose assortment B in mild weather, assortment A in hot weather, and assortment B in rainy weather. The expected value with perfect information is 74.0500. The Expected Value of Perfect Information is $74.0500 - 70.4500 = 3.6000$.

6. (*15 points*) Stanley can look at his sales of iced coffee in the morning in an attempt to predict the weather later in the day. He either sells lots of iced coffee (I) or not so much (N). Here is a table of *joint probabilities*:

	I	N
M	0.05	0.30
H	0.30	0.10
R	0.05	0.20

(a) What is $P(I)$? What is $P(N)$?

(b) If Stanley sells lots of iced coffee in the morning (I), should he load the truck with assortment A or assortment B?

(c) If Stanley sells not so much iced coffee in the morning (N), should he load the truck with assortment A or assortment B?

(d) What is the Expected Value of Stanley's strategy? Draw a decision tree to explain your reasoning.

(e) What is the efficiency of this strategy?

Answer: (a) We have $P(I) = 0.05 + 0.30 + 0.05 = 0.4000$, and $P(N) = 0.30 + 0.10 + 0.20 = 0.6000$.

(b) We compute

$$P(M|I) = 0.05/0.40 \approx 0.1250 \qquad P(H|I) = 0.30/0.40 \approx 0.7500 \qquad P(R|I) = 0.05/0.40 \approx 0.1250$$
$$P(M|N) = 0.30/0.60 \approx 0.5000 \qquad P(H|N) = 0.10/0.60 \approx 0.1667 \qquad P(R|N) = 0.20/0.60 \approx 0.3333$$

If Stanley sells lots of iced coffee in the morning, then he can compute

$$EV(A) \approx 70.1250 \qquad EV(B) \approx 52.8750$$

Therefore, if he sells lots of iced coffee in the morning, he should load the truck with assortment A.

(c) If Stanley sells not so much iced coffee in the morning, then he can compute that

$$EV(A) \approx 70.6667 \qquad EV(B) \approx 71.5000$$

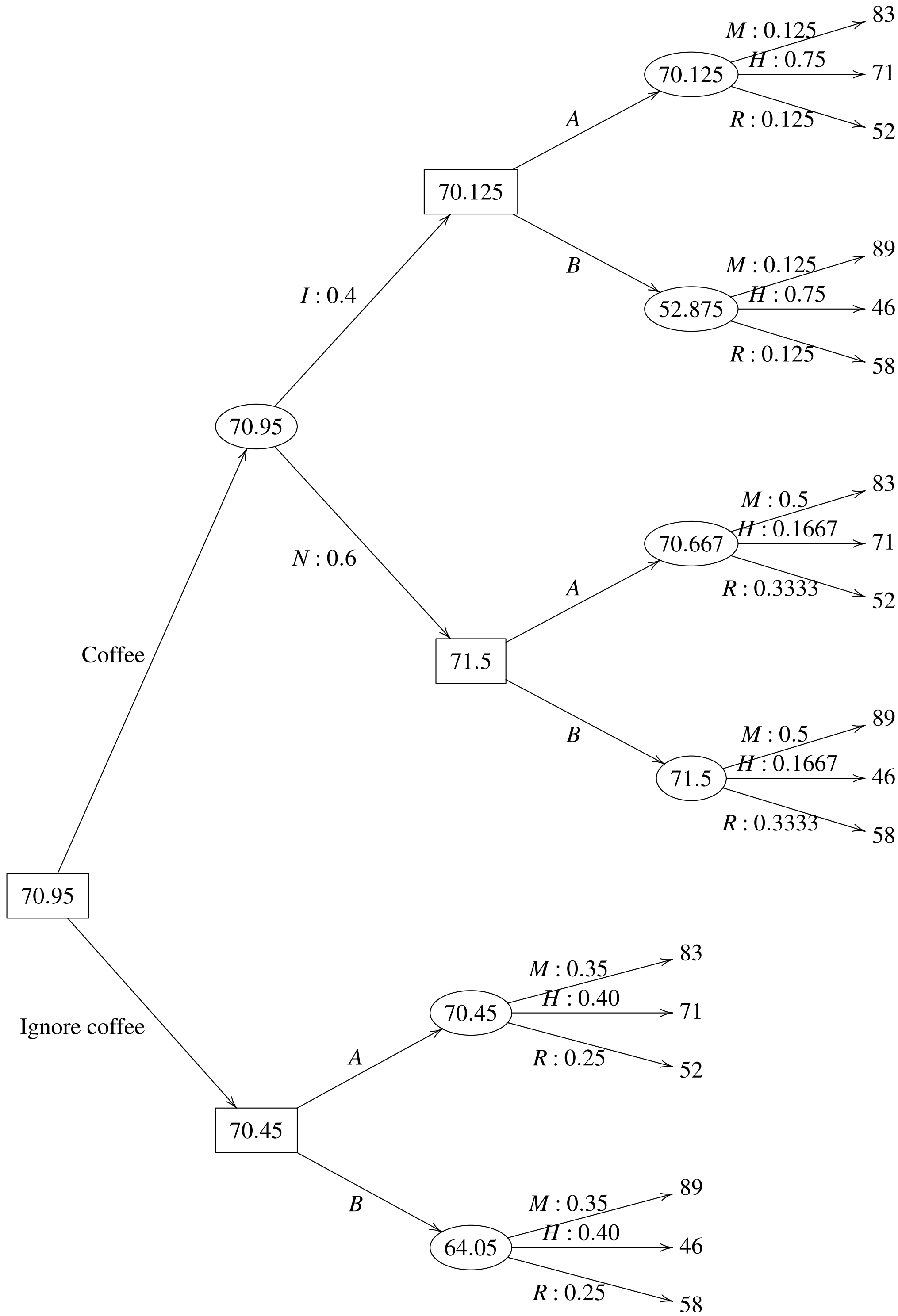
Therefore, if he sells not so much iced coffee in the morning, he should load the truck with assortment B.

(d) The expected value of this approach is $70.1250 \cdot 0.40 + 71.5000 \cdot 0.60 \approx 70.9500$.

(e) The Expected Value of Sample Information is $70.9500 - 70.4500 = 0.5000$. The efficiency is $0.5000/3.6000 \approx 0.1389 \approx 14\%$.

Grade	Number of people
100	1
95	2
90	2
89	2
86	1
85	2
82	1
68	2
65	2
62	2
53	1
27	1
23	1
10	1

Mean: 70.90
Standard deviation: 24.53



Decision Tree for Problems 5 and 6