Mathematics 235
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Homework 1
Due January 28, 2011

Homework is due at the start of class.

Please read Chapter 1 and Chapter 2, sections 1–3. You should understand the meanings of the words "slack" and "surplus."

Please remember to show all work. If your answers require more than one page, use a staple or paper clip to fasten the pages together. Do not submit crossed-out or sloppy answers, nor should you submit ripped or torn pages.

Please submit your answers to the following problems on *graph paper*. This is one of very few assignments that requires graph paper.

1. Consider the following linear programming problem:
Maximize $2x + 3y$ subject to:

$$\begin{aligned} x \phantom{{}+5y} &\le 15 \\ 2x + 5y &\le 50 \\ x + y &\le 15 \\ 3x + y &\le 35 \\ x, y &\ge 0 \end{aligned}$$

(a) Use the graphical solution procedure and the extreme points approach, find the optimal solution and the value of the objective function for that optimal solution.
(b) Check your answer to part (a) by using the iso-profit line method of finding the optimal solution.
(c) Determine the amount of slack or surplus for each constraint.
(d) Suppose that the objective function is changed to $\max(5x + 4y)$. Find the optimal solution and value of the objective function for this new problem.
(e) Suppose that the objective function is $2x + cy$. Find a value of the variable c that creates multiple optimal solutions.

2. Consider the following linear programming problem:
Minimize $4x + 5y$ subject to:

$$\begin{aligned} 2x + 3y &\ge 30 \\ x + 5y &\ge 20 \\ 2x - y &\ge 0 \\ x, y &\ge 0 \end{aligned}$$

(a) Using the graphical solution procedure and *either* the extreme points approach or the iso-profit line, find the optimal solution and the value of the objective function for that optimal solution.
(b) Determine the amount of slack or surplus for each constraint.

3. The **Oak Works** manufactures handcrafted chairs and tables from locally and sustainably grown oak. Each month, they have 2,500 pounds of oak available. Each chair uses 25 pounds of oak, and each table uses 50 pounds. Each table and each chair requires 6 hours of labor, and there are 480 hours of labor available each month. Each table has a profit of \$400, and

each chair has a profit of \$100. The company wants to produce *at least* twice as many chairs as tables.

Assume that the company is interested in maximizing the total profit contribution.

(a) Formulate a mathematical model for this problem.

(b) Find the optimal solution using the graphical solution procedure and *either* the extreme points approach or the iso-profit line. How many chairs and how many tables should be manufactured? You may assume that manufacture of fractional quantities is permissible.

(c) What is the total profit contribution with the above production quantities?

(d) Find other values for the profit of the two products that would make the optimal solution producing only chairs.

4. **Ralph** loves steak and potatoes so much that he has decided to consume only those two foods for the rest of his life. (Ralph is a stubborn teen-ager.) He is aware that this is not the healthiest possible choice of diet, so he wants to satisfy least some key nutritional requirements. He has obtained this nutrition and cost information:

Grams of Ingredient per Serving			
Ingredient	**Steak**	**Potatoes**	**Daily Requirement (grams)**
Carbohydrate	5	15	≥ 50
Protein	20	5	≥ 40
Fat	15	2	≤ 60
Cost per serving	\$4	\$2	

Ralph wishes to find the number of servings of each food (possibly fractional) to satisfy these three dietary requirements at minimal cost.

Formulate and graphically solve a mathematical model for this problem.