Mathematics 235
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Homework 2
Answers

1. Consider the following linear programming problem:
Maximize $2x + 3y$ subject to:

$$\begin{array}{rcrcr} x & & & \leq & 15 \\ 2x & + & 5y & \leq & 50 \\ x & + & y & \leq & 15 \\ 3x & + & y & \leq & 35 \\ & & x, y & \geq & 0 \end{array}$$

(a) Using *Excel* and Solver, find the optimal solution and the value of the objective function for that optimal solution.
(b) Determine the amount of slack or surplus for each constraint. Write out this part of the solution in sentences.

Answer: The answer, according to the spreadsheet, is that the maximum occurs when $x = 8.33$ and $y = 6.67$, and the value of the objective function is 36.67. The constraint $x \leq 15$ has $15 - \frac{25}{3} = \frac{20}{3} \approx 6.67$ slack. The constraint $2x + 5y = 50$ is binding. The constraint $x + y = 15$ is binding. The constraint $3x + y \leq 35$ has $\frac{10}{3} \approx 3.33$ slack.

2. Consider the following linear programming problem:
Minimize $4x + 5y$ subject to:

$$\begin{array}{rcrcr} 2x & + & 3y & \geq & 30 \\ x & + & 5y & \geq & 20 \\ 2x & - & y & \geq & 0 \\ & & x, y & \geq & 0 \end{array}$$

(a) Using *Excel* and Solver, find the optimal solution and the value of the objective function for that optimal solution.
(b) Determine the amount of slack or surplus for each constraint. Write out this part of the solution in sentences.

Answer: Excel and *Solver* report that the minimum value occurs when $x = \frac{15}{4} = 3.75$ and $y = \frac{15}{2} = 7.5$, and the value of the objective function is 52.5. The constraint $2x + 3y \geq 30$ is binding. The constraint $x + 5y \geq 20$ has a surplus of $\frac{85}{4} = 21.25$. The constraint $2x - y \geq 0$ is binding.

3. **Ralph** loves steak and potatoes so much that he has decided to consume only those two foods for the rest of his life. (Ralph is a stubborn teen-ager.) He is aware that this is not the healthiest possible choice of diet, so he wants to satisfy least some key nutritional requirements. He has obtained this nutrition and cost information:

Grams of Ingredient per Serving			
Ingredient	**Steak**	**Potatoes**	**Daily Requirement (grams)**
Carbohydrate	5	15	≥ 50
Protein	20	5	≥ 40
Fat	15	2	≤ 60
Cost per serving	\$4	\$2	

Ralph wishes to find the number of servings of each food (possibly fractional) to satisfy these three dietary requirements at minimal cost.

(a) Formulate a mathematical model for this problem.
(b) Find the optimal solution using *Excel* and Solver. How many servings of steak and how many servings of potatoes should Ralph eat each day?

Answer: Let

$$s = \text{the number of servings of steak}$$
$$p = \text{the number of servings of potatoes}$$

We need to minimize $4s + 2p$ given

Carbohydrates: $5s + 15p \geq 50$

Protein: $20s + 5p \geq 40$

Fat: $15s + 2p \leq 60$

Excel reports that the optimal solution is to eat $\frac{14}{11}$ servings of steak and $\frac{32}{11}$ servings of potatoes for a cost of approximately \$10.91.

4. **Peabody Manufacturing** has 2 production lines to make tents. Production line A has a capacity of 25 tents/hour, and production line B has a capacity of 40 tents/hour. Production line A uses 40 yards of cotton/hour and production line B uses 50 yards of cotton/hour. Each tent can be sold for \$18. Production line A is available for no more than 15 hours and production line B is available for at most 10 hours; moreover, each production line must be used for a minimum of 5 hours. There are 1000 yards of cotton available for use, at a cost of \$6/yard. In addition to the cost of the cotton, the cost of operating production line A is \$50/hour, and the cost of operating production line B is \$75/hour.

(a) Formulate a mathematical model that maximizes *profit*, meaning revenue minus all costs.
(b) Find the optimal solution using *Excel* and *Solver*. Be sure to write your solution in full sentences.

Answer: Set

$$a = \text{number of hours that production line A is used}$$
$$b = \text{number of hours that production line B is used}$$

Tent production is therefore $25a + 40b$, so revenue is $18(25a + 40b) = 450a + 720b$. The cotton usage is $40a + 50b$, so the cost of cotton is $6(40a + 50b) = 240a + 300b$. The cost of running the production lines is $50a + 75b$. Therefore, the objective function is $(450a + 720b) - (240a + 300b) - (50a + 75b) = 160a + 345b$, and we wish to maximize this function. (You can let *Excel* do this algebra for you, of course; see the posted spreadsheet.)

The constraints are:

Minimum A:	$a \geq 5$
Minimum B:	$b \geq 5$
Maximum A:	$a \leq 15$
Maximum B:	$b \leq 10$
Cotton available:	$40a + 50b \leq 1000$

Excel and *Solver* report that the optimal solution is to run assembly line A for 12.5 hours and assembly line B for 10 hours, for a total profit of \$5,450. The revenue is \$12,825, the cost of the cotton is \$6,000, and the cost of running the two production lines is \$1,375.