Mathematics 235
Robert Gross
Homework 3
Due February 11, 2011

Please note: You do not need to submit any parts of your solution this week by e-mail.

1. For the past two weeks, we have studied the following linear programming problem: Maximize $2x + 3y$ subject to:

$$\begin{aligned} x &\leq 15 \\ 2x + 5y &\leq 50 \\ x + y &\leq 15 \\ 3x + y &\leq 35 \\ x, y &\geq 0 \end{aligned}$$

(a) Draw the graph of the feasible region that you produced as part of the first homework assignment. (Either use graph paper, or else work carefully and neatly on ordinary paper.) Label the coordinates of all of the corners (vertices) of the feasible region. Label the optimal solution. You can find all of this information in earlier homework answers, so there is no credit for this part of the problem.

(b) Suppose that the objective function changes to $2x+Cy$. Fill in the blanks in the following sentence, and explain your reasoning by using graphical/algebraic sensitivity analysis:

The optimal solution will not change if $C$ varies from _____ to _____.

(c) Suppose that the objective function changes to $Dx + 3y$. Fill in the blanks in the following sentence, and explain your reasoning by using graphical/algebraic sensitivity analysis:

The optimal solution will not change if $D$ varies from _____ to _____.

(d) What is the shadow price associated with the constraint

$$x \leq 15?$$

What is the range of feasibility of this shadow price? Compute these answers using algebra and graphical methods.

(e) What is the shadow price associated with the constraint

$$2x + 5y \leq 50?$$

What is the range of feasibility of this shadow price? Compute these answers using algebra and graphical methods.

(f) What is the shadow price associated with the constraint

$$x + y \leq 15?$$

What is the range of feasibility of this shadow price? Compute these answers using algebra and graphical methods.

(g) What is the shadow price associated with the constraint

$$3x + y \leq 35?$$

What is the range of feasibility of this shadow price? Compute these answers using algebra and graphical methods.

(h) Use the *Excel* spreadsheet that you used to answer Homework 2 and produce a sensitivity report using *Solver*. Print and attach a copy of the sensitivity report, and explain which numbers in the sensitivity report correspond to the computations that you did above.

2. For the past two weeks, we have studied the following linear programming problem: Minimize $4x + 5y$ subject to:

$$\begin{aligned} 2x + 3y &\geq 30 \\ x + 5y &\geq 20 \\ 2x - y &\geq 0 \\ x, y &\geq 0 \end{aligned}$$

(a) Draw the graph of the feasible region that you produced as part of the first homework assignment. (Either use graph paper, or else work carefully and neatly on ordinary paper.) Label the coordinates of all of the corners (vertices) of the feasible region. Label the optimal solution. You can find all of this information in earlier homework answers, so there is no credit for this part of the problem.

(b) Suppose that the objective function changes to $4x+Cy$. Fill in the blanks in the following sentence, and explain your reasoning by using graphical/algebraic sensitivity analysis:

The optimal solution will not change if $C$ varies from _____ to _____.

(c) Suppose that the objective function changes to $Dx + 5y$. Fill in the blanks in the following sentence, and explain your reasoning by using graphical/algebraic sensitivity analysis:

The optimal solution will not change if $D$ varies from _____ to _____.

(d) What is the shadow price associated with the constraint

$$2x + 3y \geq 30?$$

What is the range of feasibility of this shadow price? Compute these answers using algebra and graphical methods.

(e) What is the shadow price associated with the constraint

$$x + 5y \geq 20?$$

What is the range of feasibility of this shadow price? Compute these answers using algebra and graphical methods.

(f) What is the shadow price associated with the constraint

$$2x - y \geq 0?$$

What is the range of feasibility of this shadow price? Compute these answers using algebra and graphical methods.

(g) Use the *Excel* spreadsheet that you used to answer Homework 2 and produce a sensitivity report using *Solver*. Print and attach a copy of the sensitivity report, and explain which numbers in the sensitivity report correspond to the computations that you did above.

3. Last week, we solved the following problem using *Excel*:

> **Ralph** loves steak and potatoes so much that he has decided to consume only those two foods for the rest of his life. (Ralph is a stubborn teen-ager.) He is aware that this is not the healthiest possible choice of diet, so he wants to

satisfy least some key nutritional requirements. He has obtained this nutrition and cost information:

| Grams of Ingredient per Serving | | | |
|---|---|---|---|
| **Ingredient** | **Steak** | **Potatoes** | **Daily Requirement (grams)** |
| Carbohydrate | 5 | 15 | $\geq 50$ |
| Protein | 20 | 5 | $\geq 40$ |
| Fat | 15 | 2 | $\leq 60$ |
| Cost per serving | \$4 | \$2 | |

Ralph wishes to find the number of servings of each food (possibly fractional) to satisfy these three dietary requirements at minimal cost.

Use your spreadsheet to generate a sensitivity report, and use the sensitivity report to answer the following questions. Please note that perhaps not all of the questions can be answered by using the sensitivity report; if that is the case, say so. Print out the sensitivity report and attach it to your answers.

Please note that each of these questions suggests an independent change to the problem.

(a) What happens to the optimal solution and daily cost if the daily maximum of fat is decreased to 50 grams?
(b) What happens to the optimal solution and daily cost if the cost of a serving of steak increases to \$10?
(c) What happens to the optimal solution and daily cost if the daily minimum of protein is decreased to 30 grams?
(d) What happens to the optimal solution and daily cost if the daily minimum of protein is decreased to 10 grams?
(e) What happens to the optimal solution and daily cost if the cost of a serving of potatoes increases to \$3?

4. **Oak Works** has an enormous backlog of orders for hand-made chairs and tables. The company has decided to hire three carpenters as subcontractors to take care of these orders. The three carpenters each offers an estimate how many hours it will take to build the necessary chairs and tables, along with how many hours each actually has available during the next week, and the hourly cost:

| | **Carpenter A** | **Carpenter B** | **Carpenter C** |
|---|---|---|---|
| Hours to complete all tables | 50 | 42 | 30 |
| Hours to complete all chairs | 60 | 48 | 35 |
| Hours available | 40 | 30 | 35 |
| Hourly wage | \$36 | \$42 | \$55 |

The tables tells us that the Carpenter B estimates that it would take him 42 hours to make all of the necessary tables. However, because he only has 30 hours available next week, if he only builds tables, he can only manufacture $\frac{30}{42}$ of the necessary number of orders.

(a) Formulate a linear programming model to determine what fraction of the tables and what fraction of the chairs should be built by each of the 3 carpenters so that the entire backlog can be cleared up at minimal cost.

(b) Solve your linear program using *Excel* and *Solver*. State your solution, including the final cost, in whole sentences.

Answer the next three questions using the sensitivity report produced by *Solver*. They pose three independent scenarios.

(c) Suppose that Carpenter A had 35 hours available. Say as much possible about how the solution and total cost will change.

(d) Suppose that Carpenter B had 35 hours available. Say as much possible about how the solution and total cost will change.

(e) Suppose that Carpenter C lowers his hourly rate to \$54. Say as much possible about how the solution and total cost will change.