Mathematics 235
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Homework 4
Answers

1. **Sunjuice** sells bags of orange and orange juice to wholesale companies. The oranges are graded on a scale of 1 (bad) to 10 (excellent). At present, Sunjuice has in stock 200,000 pounds of grade 9 oranges and 120,000 pounds of grade 6 oranges. The average quality of oranges sold in bags must be at least 7, and the average quality of oranges used for juice must be at least 8. Each pound of bagged oranges yields a revenue of \$1.50, while a pound of oranges can be used to make juice that sells for \$2.00. A pound of oranges used for juice has a marginal cost of \$1.05, while a pound of oranges sold in a bag has a marginal cost of \$0.70. (Both of these costs are independent of the cost of the oranges, which have already been purchased.) All of the oranges need not be used.

(a) Formulate a linear program to help Sunjuice maximize profit.

(b) Use *Excel* and *Solver* to find the optimal solution. Be sure to state your solution in full sentences, including the maximum profit.

Using the sensitivity report produced by *Solver*, answer the following two independent questions, as best as possible. It is conceivable that they cannot be answered.

(c) Suppose that 10,000 pounds of grade 6 oranges were found to be rotten and unusable. What happens to the optimal solution and maximum profit?

(d) Suppose that an additional 10,000 pounds of grade 9 oranges were found in the warehouse, at no additional cost. What happens to the optimal solution and maximum profit?

Answer: (a) Let

$$\begin{aligned} x_{b6} &= \text{number of pounds of grade 6 oranges used in bags} \\ x_{b9} &= \text{number of pounds of grade 9 oranges used in bags} \\ x_{j6} &= \text{number of pounds of grade 6 oranges used in juice} \\ x_{j9} &= \text{number of pounds of grade 9 oranges used in juice} \end{aligned}$$

We need to maximize

$$1.50(x_{b6} + x_{b9}) + 2.00(x_{j6} + x_{j9}) - 0.70(x_{b6} + x_{b9}) - 1.05(x_{j6} + x_{j9})$$

given

Grade 6 oranges available:	$x_{b6} + x_{j6} \le 120000$
Grade 9 oranges available:	$x_{b9} + x_{j9} \le 200000$
Minimum quality of bagged oranges:	$2x_{b9} \ge x_{b6}$
Minimum quality of juice oranges:	$x_{j9} \ge 2x_{j6}$

(b) *Excel* and *Solver* report that the optimal solution is to put 26,666.67 pounds of grade 6 oranges and 13,333.33 pounds of grade 9 oranges in bags, and use 93,333.33 pounds of grade 6 oranges and 186,666.67 pounds of grade 9 oranges to make juice, for a total profit of \$298000.

(c) The allowable decrease on the total quantity of grade 6 oranges is 20,000, and therefore losing 10,000 pounds is within the allowable range. The total profit would decrease by $10000 \cdot 0.65 = \$6500.00$, and the quantities of oranges used in juice and bags would change unpredictably.

(d) The allowable increase on grade 9 oranges is 40,000, so this increase is within the allowable range. The total profit would increase by $10000 \cdot 1.1 = \$11000.00$, and quantities of oranges used in juice and bags would change unpredictably.

2. **Bremer Agricultural Products** blends a silicon compound and a nitrogen compound to create two different types of fertilizer. *Mir-Gro* must be composed of at least 40% of the nitrogen compound, and sells for \$70/lb. *Super-Green* must be composed of at least 70% of the silicon compound, and sells for \$40/lb. Bremer can purchase up to 8,000 pounds of the nitrogen compound at \$15/lb, and up to 10,000 pounds of the silicon compound at \$10/lb. Bremer is confident that it can sell as much of each compound as it can produce.

(a) Formulate a linear program to help Bremer maximize profit.

(b) Use *Excel* and *Solver* to find the optimal solution. Be sure to state your solution in full sentences, including the maximum profit.

Answer: (a) Let

$$\begin{aligned} x_{nm} &= \text{number of pounds of nitrogen in Mir-Gro} \\ x_{sm} &= \text{number of pounds of silicon in Mir-Gro} \\ x_{ng} &= \text{number of pounds of nitrogen in Super-Green} \\ x_{sg} &= \text{number of pounds of silicon in Super-Green} \end{aligned}$$

We need to maximize

$$70(x_{nm} + x_{sm}) + 40(x_{ng} + x_{sg}) - 15(x_{nm} + x_{ng}) - 10(x_{sm} + x_{sg})$$

given

Mir-Gro 40% min. N:	$x_{nm} \geq 0.40(x_{nm} + x_{sm})$
Super-Green 70% min. Si:	$x_{sg} \geq 0.70(x_{sg} + x_{ng})$
8000 lb of N:	$x_{ng} + x_{nm} \leq 8000$
10000 lb of Si:	$x_{sg} + x_{sm} \leq 10000$

(b) *Excel* and *Solver* report that the optimal solution is to manufacture only Mir-Gro, using 10,000 pounds of silicon and 8,000 pounds of nitrogen, for a total profit of \$1,040,000.00.

3. **Time-Rite** manufactures clocks by hand. At present, they sell two models, with the following requirements:

	Time required		
Task	**Grandfather Clock**	**Wall Clock**	**Available**
Assembly (hrs)	6	4	40
Carving (hrs)	8	4	40
Packing/Shipping (hrs)	3	3	20
Material cost/clock	\$100	\$50	
Revenue/clock	\$500	\$300	
Maximum demand	15	20	

Time-Rite can also purchase clocks from an outside supplier. The cost of a purchased grandfather clock is \$400, and the cost of a purchased wall clock is \$150. Time-Rite can purchase a maximum of 5 clocks from the outside supplier.

(a) Formulate a linear program to help Time-Rite maximize profit.

(*b*) Use *Excel* and *Solver* to find the optimal solution. Be sure to state your solution in full sentences, including the maximum profit.

Answer: (*a*) Let

$$\begin{aligned} x_g &= \text{number of grandfather clocks made} \\ x_w &= \text{number of wall clocks made} \\ y_g &= \text{number of grandfather clocks purchased} \\ y_w &= \text{number of wall clocks purchased} \end{aligned}$$

We need to maximize

$$500(x_g + y_g) + 300(x_w + y_w) - 100x_g - 50x_w - 400y_g - 150y_w$$

given

Assembly:	$6x_g + 4x_w \leq 40$
Carving:	$8x_g + 4x_w \leq 40$
Pack/Ship:	$3x_g + 3x_w \leq 20$
Grand. dem.:	$x_g + y_g \leq 15$
Wall dem.:	$x_w + y_w \leq 20$
Outside supplier max.:	$y_g + y_w \leq 5$

(*b*) *Excel* and *Solver* report that the optimal solution is to make 3.33 grandfather clocks and 3.33 wall clocks, and purchase 5 wall clocks, for a total profit of $2,916.67.

4. As part of the settlement for a class action suit involving the sale of rotten food, **Leon's Deli Meats** must provide sufficient cash to make the following annual payments (in thousands of dollars):

Year	1	2	3	4	5	6
Payment	110	125	430	385	215	110

The annual payments must be made at the *beginning* of each year. The judge will approve an amount that, along with earnings on its investments, will cover the annual payments. Investment of the funds will be limited to savings (at 2% annually) and government securities. Both types securities have a par value (face value) of $1000, and have the following terms:

Security	**Current Price**	**Rate (%)**	**Years to Maturity**
1	$1025	5.250	2
2	$1015	4.525	3

Each security can be bought at the same terms at the start of each of the next 6 years. Funds not invested in these securities will be placed in savings. Assume that interest is paid annually at the *end* of each year. The plan will be submitted to the judge and, if approved, Leon's will be required to pay a trustee the amount that will be required to fund the plan.

Use linear programming to find the minimum cash settlement and bond purchase schedule necessary to fund the annual payments.

Answer: (*a*) Let

$$\begin{aligned} s_0 &= \text{savings at end of year } i,\ i = 0, \ldots, 5, \text{ in thousands of dollars} \\ a_i &= \text{bonds of type 1 bought at start of year } i,\ i = 1, \ldots, 4 \\ b_i &= \text{bonds of type 2 bought at start of year } i,\ i = 1, \ldots, 3 \end{aligned}$$

We need to minimize s_0, given that

Year 1: $s_0 - 110 - 1.025a_1 - 1.015b_1 = s_1$

Year 2: $1.02s_1 - 125 - 1.025a_2 - 1.015b_2 + 0.05250a_1 + 0.04525b_1 = s_2$

Year 3: $1.02s_2 - 430 - 1.025a_3 - 1.015b_3 + 0.05250(a_1 + a_2) + 0.04525(b_1 + b_2) + a_1 = s_3$

Year 4: $1.02s_3 - 385 - 1.025a_4 + 0.05250(a_2 + a_3) + 0.04525(b_1 + b_2 + b_3) + a_2 + b_1 = s_4$

Year 5: $1.02s_4 - 215 + 0.05250(a_3 + a_4) + 0.04525(b_2 + b_3) + a_3 + b_2 = s_5$

Year 6: $1.02s_5 - 110 + 0.05250a_4 + 0.04525b_3 + a_4 + b_3 \geq 0$

(b) *Excel* and *Solver* report that the optimal solution is to start with \$1,247,019.22, purchase 582.60 bonds of type 1 and 460.82 bonds of type 2 in year 1, and also purchase 199.06 bonds of type 1 in year 3 and 104.51 bonds of type 1 in year 4.