Mathematics 235
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Homework 5
Answers

1. **Bronco Lawn Mowers** manufactures a gasoline and an electric model. Demand for April, May, and June, along with maximum production for each month, are:

	Demand			Capacity		
	April	May	June	April	May	June
Gasoline	680	720	900	800	800	800
Electric	500	600	700	650	650	650

All demand must be met. The monthly cost of storing a mower in inventory is \$15. Changing total production from month to month costs \$20 for each additional mower manufactured, and \$10 for each unit decrease. The production levels in March are 600 gasoline mowers and 700 electric mowers. No mowers are currently in inventory, and none need to be in inventory at the end of June.

(*a*) Formulate a linear program to help Bronco minimize the costs detailed above. You may assume that all other costs are fixed.

(*b*) Use *Excel* and *Solver* to find the optimal solution. Be sure to state your solution in full sentences, including the minimum cost.

Answer: Number the months so that March=0, April=1, May=2, and June=3. Let

$$\begin{aligned}
g_i &= \text{Gasoline mowers produced in month } i,\ i = 1,2,3 \\
e_i &= \text{Electric mowers produced in month } i,\ i = 1,2,3 \\
x_i &= \text{Gasoline mowers in inventory at end of month } i,\ i = 1,2,3 \\
y_i &= \text{Electric mowers in inventory at end of month } i,\ i = 1,2,3 \\
I_i &= \text{Production increase from month } i-1 \text{ to month } i,\ i = 1,2,3 \\
D_i &= \text{Production decrease from month } i-1 \text{ to month } i,\ i = 1,2,3
\end{aligned}$$

We need to minimize

$$15(x_1 + x_2 + x_3 + y_1 + y_2 + y_3) + 20(I_1 + I_2 + I_3) + 10(D_1 + D_2 + D_3)$$

given

$$\begin{aligned}
&\text{April capacity:} & g_1 &\le 800 \\
& & e_1 &\le 650 \\
&\text{May capacity:} & g_2 &\le 800 \\
& & e_2 &\le 650 \\
&\text{June capacity:} & g_3 &\le 800 \\
& & e_3 &\le 650 \\
&\text{March-April production change:} & 600 + 700 + I_1 - D_1 &= g_1 + e_1 \\
&\text{April-May production change:} & g_1 + e_1 + I_2 - D_2 &= g_2 + e_2 \\
&\text{May-June production change:} & g_2 + e_2 + I_3 - D_3 &= g_3 + e_3 \\
&\text{April flow:} & g_1 &= 680 + x_1 \\
& & e_1 &= 500 + y_1 \\
&\text{May flow:} & g_2 + x_1 &= 720 + x_2 \\
& & e_2 + y_1 &= 600 + y_2 \\
&\text{June flow:} & g_3 + x_2 &= 900 + x_2 \\
& & e_3 + y_2 &= 700 + y_2
\end{aligned}$$

The solution, according to *Excel* and *Solver*, is to manufacture 700, 800, and 800 gas mowers in the next three months, 600 electric mowers in each of the next three months, put 20 gas mowers in inventory in April and 100 in May, and put 100 electric mowers in inventory in both April and May. The total cost is \$9,800.

2. **Luigi's Soups** comes in three varieties: tomato, minestrone, and chicken. Each soup needs to be cooked, cooled, and canned, according to the following table:

	Tomato	Minestrone	Chicken	Minutes available
Cooking	12	14	18	2500
Cooling	4	3	4	2000
Canning	2	1	1	1200

The profits on a can of each soup are \$0.80, \$0.74, and \$0.31, respectively.

(*a*) Formulate a linear program to help Luigi maximize his profit.

(*b*) Use *Excel* and *Solver* to find the optimal solution. Be sure to state your solution in full sentences, including the maximum profit.

(*c*) The optimal solution does not call for manufacturing all three types of soup. Use the sensitivity report to decide at what price(s) it might be worthwhile producing the soup(s) that are not part of the optimal solution.

Luigi notices that he has time available in one or more departments, and negotiates with his workers. For an additional fee of \$3/hr, he can transfer time from any department to any department.

(*d*) Add decision variables as needed and alter the objective function as needed to formulate a new linear program for the modified problem.

(*e*) Use *Excel* and *Solver* to find the optimal solution. Be sure to state your solution in full sentences, including the maximum profit.

Answer: Let

$$\begin{aligned} t &= \text{number of cans of tomato soup manufactured} \\ m &= \text{number of cans of minestrone manufactured} \\ c &= \text{number of cans of chicken soup manufactured} \end{aligned}$$

We need to maximize $0.80t + 0.74m + 0.31c$. The constraints are

$$\begin{aligned} &\text{Cooking:} & 12t + 14m + 18c &\le 2500 \\ &\text{Cooling:} & 4t + 3m + 4c &\le 2000 \\ &\text{Canning:} & 2t + m + c &\le 1200 \end{aligned}$$

Excel reports that the optimal solution is to manufacture 208.333 cans of tomato and none of the other types of soup, for a maximum profit of \$166.67.

The sensitivity report says that the reduced cost for minestrone is -0.19333, so if the profit on minestrone increased to $0.74 + 0.19333 = 0.9333$, then minestrone might be worth producing. Similarly, the reduced cost of chicken soup is -0.89, meaning that the profit on chicken soup would have to increase to $0.31 + 0.89 = 1.20$ in order for chicken soup to potentially be worth producing.

(*d*) Number the departments 1, 2, and 3 respectively. Let x_{ij} be the number of *hours* transferred from department to department. The objective function becomes $0.80t + 0.74m + 0.31c - 3(x_{12} + x_{13} + x_{21} + x_{23} + x_{31} + x_{32})$. The new constraints are:

$$\begin{aligned} &\text{Cooking:} & 12t + 14m + 18c &\le 2500 + 60(x_{21} + x_{31} - x_{12} - x_{13}) \\ &\text{Cooling:} & 4t + 3m + 4c &\le 2000 + 60(x_{12} + x_{32} - x_{21} + x_{23}) \\ &\text{Canning:} & 2t + m + c &\le 1200 + 60(x_{13} + x_{23} - x_{31} - x_{32}) \end{aligned}$$

The optimal solution is still to produce only tomato soup, but now 316.67 cans of tomato soup, for a profit of \$188.33. We transfer 12.22 hours from cooling to cooking, and 9.44 hours from canning to cooking.

3. **Omaha Coffee** blends four types of beans to make two different house blends. The four types are light-roasted arabica, dark-roasted arabica, light-roasted robusta, and dark-roasted robusta, and the two different house blends are Mellow-Glow and Espresso. Mellow-Glow can be no more than 50% dark roast, and no more than 20% robusta. Espresso must be at least 80% dark roast, and no more than 30% robusta. The costs and maximum availability of each of the 4 types of beans are

	Cost/lb	Pounds available
Light-roast arabica	\$1.10	2300
Dark-roast arabica	\$1.30	2500
Light-roast robusta	\$0.82	4400
Dark-roast robusta	\$0.88	2300

A pound of Mellow-Glow sells for \$12, and a pound of Espresso sells for \$14.

(a) Formulate a linear program to help Omaha Coffee maximize profit.

(b) Use *Excel* and *Solver* to find the optimal solution. Be sure to state your solution in full sentences, including the maximum profit.

(c) Suppose that an additional 100lb of light-roast arabica were available at a cost of \$5/lb. Is this a wise purchase?

(d) Omaha is contemplating adding a new blend, to consist of equal parts of light-roast and dark-roast arabica. The anticipated sales price of the new blend is \$15/lb. Using only the sensitivity report, can you determine if this product should be considered further?

Answer: Let

$$\begin{aligned}
m_{la} &= \text{number of pounds of light-roast arabica in Mellow-Glow}\\
m_{da} &= \text{number of pounds of dark-roast arabica in Mellow-Glow}\\
m_{lr} &= \text{number of pounds of light-roast robusta in Mellow-Glow}\\
m_{dr} &= \text{number of pounds of dark-roast robusta in Mellow-Glow}\\
e_{la} &= \text{number of pounds of light-roast arabica in Espresso}\\
e_{da} &= \text{number of pounds of dark-roast arabica in Espresso}\\
e_{lr} &= \text{number of pounds of light-roast robusta in Espresso}\\
e_{dr} &= \text{number of pounds of dark-roast robusta in Espresso}
\end{aligned}$$

We wish to maximize

$$12(m_{la}+m_{da}+m_{lr}+m_{dr})+14(e_{la}+e_{da}+e_{lr}+e_{dr})$$
$$-$$
$$\Big(1.10(m_{la}+e_{la})+1.30(m_{da}+e_{da})+0.82(m_{lr}+e_{lr})+0.88(m_{dr}+e_{dr})\Big)$$

given

Light-roast arabica:	$e_{la}+m_{la}\le 2300$
Dark-roast arabica:	$e_{da}+m_{da}\le 2500$
Light-roast robusta:	$e_{lr}+m_{lr}\le 4400$
Dark-roast robusta:	$e_{dr}+m_{dr}\le 2300$
Mellow-roast dark maximum:	$m_{dr}+m_{da}\le 0.50(m_{dr}+m_{da}+m_{lr}+m_{la})$
Mellow-roast robusta maximum:	$m_{dr}+m_{lr}\le 0.20(m_{dr}+m_{da}+m_{lr}+m_{la})$
Espresso dark minimum:	$e_{dr}+e_{da}\ge 0.80(e_{dr}+e_{da}+e_{lr}+e_{la})$
Espresso robusta maximum:	$e_{dr}+e_{lr}\le 0.30(e_{dr}+e_{da}+e_{lr}+e_{la})$

Excel and *Solver* report that the optimal solution is to blend 1625 pounds of Mellow-Glow from 1300 pounds of light-roast arabica and 325 pound of light-roast robusta, and 5000 pounds of espresso from 1000 pounds of light-roast arabica, 2500 pounds of dark-roast arabica, and 1500 pounds of dark-roast robusta, for a total profit of \$82,133.50.

The shadow price of light-roast arabica is 13.695, and the allowable increase is much larger than 100. Every additional pound of arabica will increase our profit by that \$13.695. If the cost is only \$5/lb, this purchase is a good one.

The shadow price of dark-roast arabica is 20.254. Decreasing the supply of light and dark-roast arabica by a half-pound each will decrease our profit by $0.5(13.695 + 20.254) = 16.9745$. This is more than \$15, so the new product will not increase profits and should not be manufactured.