Mathematics 235
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Homework 8
Due April 8, 2011

1. **Burnside Marketing Research** conducted a study to design a new breakfast cereal. Three attributes were found to be significant: ratio of wheat to corn, type of sweetener (sugar, honey, or artificial), and the presence or absence of raisins. Seven children participated in taste tests and provided the following point values for their preferences:

	Wheat/Corn		**Sweetener**			**Raisins**	
Child	**Low**	**High**	**Sugar**	**Honey**	**Artificial**	**Present**	**Absent**
1	15	35	30	40	25	15	9
2	30	20	40	35	35	8	11
3	40	25	20	40	10	7	14
4	35	30	25	20	30	15	18
5	25	40	40	20	35	18	14
6	20	25	20	35	30	9	16
7	30	15	25	40	40	20	11

(*a*) Formulate a linear model that produces a cereal recipe with the highest average score among the seven children.

(*b*) Solve using *Excel* and *Solver*. What is the cereal recipe with the highest average score?

(*c*) The current leading brand has a **low** ratio of wheat to corn, is sweetened with **honey**, and does **not** have raisins. Formulate a linear model for a cereal that will score at least 1 point higher with as many children as possible. The objective function is the number of children who will prefer the new cereal to the current leading brand.

(*d*) Solve your model using *Excel* and *Solver*. What is the cereal recipe that will induce the maximum number of children to change brands?

(*e*) Change the spreadsheet to ask for a score that will be at least 5 points higher than the current leading brand. Solve this reformulated problem. What is the recipe for the new cereal that will have a score at least 5 points higher with as many children as possible?

2. **Green's Hardware** expects to sell 75,000 bolts annually. The cost of ordering bolts is \$15. Inventory cost is \$0.01/bolt/year. Suppose that the unit cost is \$0.15/bolt. What order size minimizes Green's annual costs? Formulate and solve using calculus. You need not assume that the order size is an integer. You might want to check your answer using *Excel* and *Solver*, but you need not submit that part of your solution.

3. Suppose that $f(x) = 3x^4 + 4x^3 - 36x^2 + 5$.

(*a*) Find all critical values of $f(x)$, and use the second derivative test to determine whether a critical value is a local maximum or a local minimum.

(*b*) Suppose that we impose the restriction that $1 \leq x \leq 5$. Find the largest and smallest values that $f(x)$ can take.

4. **Tremont Concrete** orders 20,000 pounds of sand annually. The cost of placing an order is \$20. Inventory cost is 1%/lb/year of the unit cost of a pound of sand.

(*a*) Suppose that the cost of sand is \$1.30/lb. What order size minimizes Tremont's total cost? Solve using calculus. Orders need not be for a whole number of pounds.

(*b*) Suppose now that there is a quantity discount. The price is \$1.30/lb on orders of at most 5,000 pounds, but on orders of 5,001 pounds or more, the price is \$1.20/lb. To clarify, this means that the price of ordering 5,001 pounds is (\$1.20)(5,001)=\$6,001.20. Now find the order size that minimizes Tremont's total cost. Again, minimize using calculus.

(*c*) More realistically, suppose that the quantity discount works so that the first 5,000 pounds of any order cost \$1.30/lb, and any additional pounds of sand cost \$1.20. Now an order of 5,001 pounds costs (\$1.30)(5,000) + (\$1.20)(1) = \$6,501.20. What order size minimizes total cost?

5. Let $f(x, y) = 2x^2 + 3xy + y^2 - 11x - 8y + 14$. Find all critical pairs for $f(x, y)$, and use the second derivative test to classify each critical pair as a local minimum, a local maximum, or a saddle point.