Mathematics 235
Robert Gross
Homework 9
Due April 15, 2011

1. A certain port has 3 loading docks, labelled A, B, and C. The distances between the docks, in meters, is:

	A	B	C
A	–	100	175
B	100	–	75
C	175	75	–

Three boats, the *Nina*, the *Pinta*, and the *Santa Maria*, need to come into port and dock simultaneously, one at each dock, and transfer goods. The quantity of cargo to be transferred, in kilograms, is

	To		
From	*Nina*	*Pinta*	*Santa Maria*
Nina	–	60	75
Pinta	–	–	80
Santa Maria	35	85	–

The objective is to minimize the product of the weight of the cargo that must be moved and the distance that the cargo is moved. For example, if the *Nina* is in dock A, the *Pinta* in dock B, and the *Santa Maria* in dock C, then the product will be $60 \cdot 100 + 75 \cdot 175 + 80 \cdot 75 + 175 \cdot 35 + 75 \cdot 85$.

Formulate this problem, which probably will involve binary variables. Be sure to state your objective function and constraints clearly. The objective function will probably not be linear. Then use *Excel* and *Solver* to find the optimal solution.

2. The Cobb–Douglas production function is a clssic model from economics used to model output as a function of capital and labor. It has the form

$$f(L, C) = aL^b C^d$$

where L represents units of labor, C represents units of capital, and a, b, and d are constants.

For example, set $a = 5$, $b = 0.25$ and $d = 0.75$, and assume that the cost of a unit of labor is \$25 and the cost of a unit of capital is \$75. Assume that a total of \$75,000 is available, and will all be used for either labor or capital.

(a) Write out a model to maximize $f(L, C)$. Be sure to state your constraints clearly. The model will be non-linear.
(b) Use *Excel* and *Solver* to find the optimal solution.
(c) What is the approximate shadow price for the constraint that a total of \$75,000 is available?

3. Suppose that we need to find both the maximum and the minimum of the function $f(x, y) = 2x+3y$, given the constraint $x^2 + y^2 = 10$.

(a) Formulate this problem using Lagrange multipliers. Write out the 3 equations involving x, y, and λ. Then solve the equations.
(b) Solve the problem using *Excel* and *Solver*. You will need to run *Solver* twice, once to find the maximum and once to find the minimum. Do not constraint the variables x and y to be positive.
(c) Verify that the numerical solutions found by *Solver* are the same as the ones that you found in part (a).

4. **Western Sports** sells winter clothing. The store must place its order for down vests well before the season starts, because they are made during the summer. Western must decide whether to place a large, medium, or small order for vests, and the number sold will depend mostly on whether the winter is mild, normal, or frigid.

The following table summarizes the net profit (in thousands of dollars) that Western expects under each scenario:

	Amount of Snow		
Size of order	Frigid	Normal	Mild
Large	10	7	3
Medium	9	9	7
Small	4	4	4

(*a*) Using the optimistic decision rule, what decision should Western Sports make? Explain your answer.

(*b*) Using the conservative decision rule, what decision should Western Sports make? Explain your answer.

(*c*) Use the minimax regret rule to help Western Sports decide. Explain your answer.

(*d*) Suppose that the probability of a frigid winter is 0.2, that of a normal winter is 0.5, and that of a mild winter is 0.3. Use the expected value approach to formulate a decision. What is the Expected Value of Perfect Information?

5. **Grow-It Farms** has a choice of four crops to plant in a certain field this month. The farm manager estimates the following yield in bushels, and incomes for each of the four crops, depending on the weather:

	Expected Yield			
Weather	**Crop 1**	**Crop 2**	**Crop 3**	**Crop 4**
Dry (D)	20	15	30	40
Moderate (M)	35	20	25	40
Rainy (R)	40	30	25	40
Income/bushel	\$1.00	\$1.50	\$1.00	\$0.80

(*a*) Using the optimistic decision rule, what decision should Grow-It make? Explain your answer.

(*b*) Using the conservative decision rule, what decision should Grow-It make? Explain your answer.

(*c*) Use the minimax regret rule to help Grow-It decide. Explain your answer.

(*d*) Suppose that the probability of a dry weather (D) is 0.10, that of moderate weather (M) is 0.55, and that of a rainy weather (R) is 0.35. Use the expected value approach to formulate a decision. What is the Expected Value of Perfect Information?

(*e*) Suppose that Grow-It can wait a week, and use the weather for that week to help estimate the various probabilities for the upcoming growing season. In particular, Grow-It will classify the rainfall during the coming week as high (H) or low (L), and use the following probabilities:

$$P(H) = 0.5300 \qquad P(L) = 0.4700$$
$$P(D|H) = 0.0189 \qquad P(D|L) = 0.1915$$
$$P(M|H) = 0.5189 \qquad P(M|L) = 0.5851$$
$$P(R|H) = 0.4623 \qquad P(R|L) = 0.2234$$

If Grow-It waits a week, what strategy should be followed? What is the Expected Value of Sample Information? What is the efficiency of the survey?