

Mathematics 235  
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 Homework 10  
 Due April 29, 2011

1. **Joe Plutocrat** has been approached by 4 hedge funds with 4 different plans to minimize his taxes. The unknown state of nature is a combination of what the tax law will be next year, and how the economy will perform. Mr. Plutocrat estimates that there are 3 possible states of nature after considering the possibilities:

- (a) Taxes will be levied at a high rate on all assets valued at more than \$1,000,000 and the economy will grow because the deficit decreases.
- (b) Tax rates will be lower, and the economy will stagnate.
- (c) Tax rates will be lower, and the economy will soar.

Mr. Plutocrat computes that the net payoff, after expenses (in millions of dollars) from each of the four hedge funds in each of these three cases is:

Hedge Fund	State of nature		
	(a)	(b)	(c)
A	71	58	91
B	92	83	38
C	62	42	67
D	23	98	93

- (a) Using the optimistic decision rule, what decision should Mr. Plutocrat make? Explain your answer.
- (b) Using the conservative decision rule, what decision should Mr. Plutocrat make? Explain your answer.
- (c) Use the minimax regret rule to help Mr. Plutocrat decide. Explain your answer.
- (d) Mr. Plutocrat hires a consultant who says that he can predict the state of nature based on whether the stock market is heading up ( $U$ ) or down ( $D$ ). He says that  $P(U) = 0.4050$  and  $P(D) = 0.5950$ . Moreover, he reports that  $P(a|U) = 0.3951$ ,  $P(b|U) = 0.4321$ , and  $P(c|U) = 0.1728$ , and that  $P(a|D) = 0.4034$ ,  $P(b|D) = 0.1261$ , and  $P(c|D) = 0.4706$ .

Using this information, what are  $P(a)$ ,  $P(b)$ , and  $P(c)$ ? In the absence of information about whether the market will head up or down, what does the Expected Value approach tell Mr. Plutocrat to do? What is the EVwoPI, the EVwPI, and the EVPI?

- (e) What should he do if the stock market heads up? What should he do if the market heads down? What is the expected value of the strategy that uses the stock market's activity?

2. A theatrical producer earns an average of \$1.6 million from a hit musical, and loses an average of \$400,000 from a flop. In recent years, 25% of his productions have been hits, and 75% flops. If the producer knew in advance which shows would flop, he would decline to stage them.

A market research company offers to help the producer identify hit musicals. Their past history shows that if a show is actually going to be a hit, they will predict that it will be a hit 90% of the time. If a show will flop, they incorrectly predict that it will be a hit 25% of the time.

- (a) What is the Expected Value without Perfect Information (EVwoPI)? What is the Expected Value with Perfect Information? What is the Expected Value of Perfect Information?
- (b) How much money should the producer be willing to pay the market research firm? In other words, what is the Expected Value of Sample Information?
- (c) What is the efficiency of the market research?

3. A bank must decide whether to extend a \$100,000 line of credit to a company which has applied for a loan. Past experience with other firms in the same industry shows that 45% of these loans are high risk and lose \$60,000; 35% are moderate risk and make \$50,000; and 20% of the loans are low risk, and eventually result in a profit of \$100,000. If the bank does not extend the line of credit, there is neither a loss nor a gain.

(a) In the absence of any other information, what is the Expected Value of extending the line of credit?

(b) What is the Expected Value of Perfect Information?

A credit rating agency charges \$2,000 for a report which is not completely reliable. The agency's success rate is:

<i>Actual rating</i>	<i>Agency's Rating</i>		
	Low	Medium	High
Low	0.85	0.10	0.05
Medium	0.06	0.87	0.07
High	0.01	0.05	0.94

(c) If the credit agency's report is consulted, what strategy should be followed?

(d) What is the Expected Value of Sample Information, including the agency's fee?

(e) What is the efficiency of the survey? This computation should not include the agency's fee.

4. To save on expenses, Rona and Jerry agreed to form a carpool for traveling to and from work. Rona preferred to use the somewhat longer but more consistent Queen City Avenue. Although Jerry preferred the quicker expressway, he agreed with Rona that they should take Queen City Avenue if the expressway had a traffic jam. The following payoff table provides the one-way time estimates in minutes for traveling to or from work:

<i>Decision Alternative</i>	<i>State of nature</i>	
	Expressway open ( $s_1$ )	Expressway jammed ( $s_2$ )
Queen City Avenue, $d_1$	30	30
Expressway, $d_2$	25	45

Based on their experience with traffic problems, Rona and Jerry agreed on a 0.15 probability that the expressway will be jammed.

In addition, they agreed that weather seemed to affect the traffic conditions on the expressway. Let

$C$  = clear

$O$  = overcast

$R$  = rain

The following conditional probabilities apply:

$$P(C|s_1) = 0.8 \quad P(O|s_1) = 0.2 \quad P(R|s_1) = 0.0$$

$$P(C|s_2) = 0.1 \quad P(O|s_2) = 0.3 \quad P(R|s_2) = 0.6$$

(a) Use the Bayes' probability revision procedure to compute the probability of each weather condition and the conditional probability of the expressway open  $s_1$  or closed  $s_2$  given each weather condition.

(b) Show the decision tree for this problem.

(c) What is the optimal decision strategy and what is the expected travel time?